INSPECTION ERRORS AND SAMPLING PLANS

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ISHWAR B. HEMRAJANI

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CERTIFICATE

Certified that the work on 'Inspection Errors and Sampling Plans', by Mr. Ishwar B. Hemrajani, has been carried out under my supervision and has not been submitted elsewhere for a degree.

(Kripa Shanker)
Assistant Professor
Industrial and Management Engg.
Indian Institute of Technology
Kanpur

July, 1982

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ABSTRACT

The present work on inspection errors deals with the effect of inspection errors on various statistical measures of some sampling plans. Analysis related to the design of plans compensating for the presence of errors is also presented.

The single, double and sequential sampling plans are considered for the attributes case. The effect of inspection errors on these plan is studied. It is found that performance measures such as probability of acceptance and average outgoing quality etc. are significantly affected, in the presence of errors. Compensating design procedures have also been discussed.

In the context of variables, single and sequential sampling plans have been studied, in the presence of measurement error. Effect of errors on probability of acceptance is analyzed and a design procedure is discussed to compensate for the presence of measurement errors.

The effects of inspection errors, both on the statistical and economic measures suggest that there is a need to incorporate inspection errors in the model building phase of quality control if quality control systems are to be accurately represented.

CHAPTER I

INTRODUCTION

1.1 Human Aspects, Quality and Quality Control:

1.1.1 Human Aspects:

Technology is effective to the extent that men can operate and maintain the machines they design. Equipment design which consciously takes into account, advantages of human capabilities and constraints itself within human limitations would prove to be the most economic design. This consideration is more important in complex systems being developed. The present systems are pressing human functions more and more towards the limits of the efficient performance. The concepts of 'Human Aspects in Engineering' can be logically extended to 'Quality and Quality Control'. Let us first consider, what is Quality? and what is Quality Control?

1.1.2 Quality and Quality Control:

All human institutions (industrial organisations, hospitals, governments, etc.) exist to provide products or services to human beings. An essential aspect of these products and services is that they must be fit for use. The phrase 'fitness for use' is the basic meaning of 'Quality' [Juran et al (15)]. Quality has long been recognized as a very desirable property of the manufactured product.

Quality control is an integral part of any manufacturing activity. It is an effective system for integrating the quality development, the quality maintenance and the quality improvement efforts of various groups in the organisation so as to enable the production and service at most economical levels which allow for full customer satisfaction [Feigenbaum (6)].

W.E. Demig (14) has defined Statistical Quality Control as follows. Statistical quality control is the application of statistical principles and techniques in all stages of production, directed towards the economic manufacture of a product that is maximally useful and has a market.

1.1.3 Human Aspects and Quality Control:

The human performance was obviously important in the past, when most of the products were hand made. Today with rapid advancement of automation, human performance is, in a way, loosing its importance. The importance of people is evident in both, the production process in which human errors may lead to the defective products and in quality control where human errors may lead to inaccurate judgement and information resulting in ineffective problem solving.

People can be viewed as critical element in the socio-technical system. The people, equipment, material and the information organize together to ensure that the products are made to certain standards of quality. These three may be

thought of as basic entities of a quality system. According to observations made by Harris et al (10), there has been plenty of evidence to indicate that people play important role in determining the product quality and reliability. Similar observations have been reported by Drury et al (4).

People conceive and design new products and processes, people select the material, parts and equipment required for the new products, people assemble, inspect and test. Finally, people pack, deliver and service these products. In performing these functions, each task is subjected to various types of human errors.

Basically human performance affecting the quality is through the operator and the inspector. The operator is, in a way element of the production system. Inspector is therefore considered as the sole contributor of human-errors in quality control system. Hereafter, terms human errors, inspection errors or simply errors will be used to denote the same.

1.1.4 Assumption of Perfect Inspection:

Classically all quality control models assume that inspection is perfect. In the case of variables being measured on a continuous scale not only inspector but even measuring instruments are assumed to be absolutely accurate. Hence it is implicitly assumed that the dimension being measured is free from measurement error and hence the measured

value is true representation of the quality characteristic. But unfortunately there is no single instrument which will give absolutely accurate results. This along with human limitations give rise to what may be termed as measurement errors.

In the same way, sampling inspection plans for attributes have implicit assumption of perfect inspector. This assumption is of course not true. In reality an inspector would be making two types of errors. He may be classifying a good item as defective (type I error) or/and may be classifying a defective item as good (type II error). Although inspection errors are unintentional, they nevertheless can severely distort the objectives of a quality control system, which has ignored their presence.

1.2 Three Facets of Human Aspects:

The subject of human factors engineering has a small subset, which may be termed as 'human factors in quality control'. The research in the field of human factors in quality control can be divided into following three categories.

(i) Dealing with the factors that affect the inspection performance and other psychological aspects.

(ii) Dealing with measurement of the inspection performance.

(iii) Dealing with the analysis of quality control models in the presence of inspection errors.

Each one of the three is dependent on the other two. The first category is of chief interest to psychologists. However, the outcome of their research may be extremely useful for the quality control system. This area has been fairly well investigated and several experimental results have been obtained [Harris et al (10), Drury et al (4)].

The work done in this area is of the following type. How does the nature of job affect the performance, what is the effect of defect rate ? How inspection tools and techniques etc. affect the performance? It includes developing guidelines for selection and training of the inspector. Quality motivation is another area of interest.

Another facet of human aspects research in quality control includes the measurement of inspectors performance and comparative study of performance measures. This is the area of interest to the ergonomists. They probably act as a link between the psychologists and quality control engineers. The simplest of performance measures are, the probabilities of committing type I error and type II errors (these are represented by e_1 and e_2). The others measures includes specificity and sensitivity, measures based on theory of signal detection etc. specificity is probability that a non-defective is classified as such (i.e. $1 - e_1$). Sensitivity is probability that a defective item is classified correctly (i.e. $1 - e_2$). There are quite a few performance measures

but these will not be discussed, this area not being of our primary interest.

Just to mention at this stage, in the case of variables being measured on a continuous scale, the performance measures are bias (μ_e) and imprecision (σ_e). These measures will be discussed in Chapter V.

The third important category is the analysis of quality control models in presence of errors. This is the main aim of the work at hand.

Basically there is an implicit assumption that inspection for acceptance sampling or in the case of measurement is error free. We will try to relax this assumption. In case of attributes, e_1, the probability of committing type I error (classifying a good item as defective) and e_2, the probability of commiting type II errors (classifying a defective as good) will be considered as pair of performance measures. How the presence of inspection error (in terms of e_1 and e_2) affect the analysis of sampling plan and what are their cost implications will be the main interests to work upon. In the case of variables sampling, the effect of bias (μ_e) and imprecision (σ_e) on some of the models will be studied.

1.3 Literature Survey:

Although the study of the effects of inspection errors on sampling plans is fairly recent, still considerable amount of work has been done.

Considerable work has been done in the area of psychological aspects of human factors. Harris et al (10) have condensed few of the experimental results in form of a book. Drury et al (4) have also described in detail about these aspects. Inspection accuracy, factors affecting the human performance etc. are the titles, which we would frequently come across in the literature.

Johnson et al (20) and Teimstra (33) have discussed various performance measures and their comparative study. Baker (37) has based his work upon theory of signal detection. These are some of the contributionsin the category of, measuring inspection.

Case, Benett, Collins, Schmidt (17,23,24,25,27) have done the major work in the third category i.e. the analysis of quality control models in the presence of inspection errors. They have discussed the effect of errors on various performance measures of a single sampling plan. Other notable contributions are by Ayoub et al (21, 22), Beainy et al (16). The later have tried to discuss the effect of errors on average outgoing quality (AOQ) and average total inspection (ATI) for nine different rectification policies given by Worthram et al (27).

The 'p' control chart under inspection errors has been discussed by Case (18). Continuous sampling plan CSP-1 is discussed by Case et al (26).

Inspection errors will depend upon several factors including the incoming quality. Beigal (30) considers the inspection error to be a linear function of the incoming quality and analyzes the effect on the performance measures of a single sampling plan.

The economic model has been discussed by Case et al (27). In brief, considerable amount of work has been done for the case of single sampling plan. Main aim of the present volume is to extend the concepts for the cases of double and sequential sampling plans.

In case of variables acceptance sampling, Mei et al (34) have discussed the effect of measurement errors on the probability of acceptance and have given a compensating design procedure. Case et al (35) have provided an economic model analysing the economic effects of the measurement errors.

1.4 Organization of Thesis:

The study of the effect of inspection errors on various performance measures of sampling plans and modification of the models to compensate for the presence of errors forms the core of the thesis.

In Chapter II, the single sampling plan is considered. Single sampling plan is probably one of the most extensively used tools of statistical quality control. The effect of

inspection errors on the performance measures of single sampling plan is first discussed. A model is then presented to compensate for the presence of inspection errors, when designing the plan on the basis of quality protection requirements i.e. (AQL, α), (RQL, β) requirements. A cost model is discussed to have an idea of the adverse cost effects of errors and the economic design of a single sampling plan in presence of inspection errors. Analysis of AOQL and ATI based plan is also presented.

The effect of inspection errors on single sampling plan is a fairly well investigated area but still occupies a large space in this work, primarily because single sampling plan is a basic tool and analysis of this helps in developing few important concepts.

The Chapter III is attributed to the analysis of double sampling plan in the presence of inspection errors. The analysis proceeds in the same way as in Chapter II. First of all, the performance measures of double sampling plan are analysed in the presence of inspection errors. A model is then presented to design a double sampling which accounts for the presence of inspection errors. Again the design is based on quality protection requirements. An exact economic analysis of double sampling plan in presence of errors is also presented.

The Chapter IV considers the sequential sampling plan. Here again the first step is to analyse the effect of inspection errors on the performance measures of the plan. A model is then presented to design the plan based on quality protection requirements and compensating for the inspection errors present. A comparative study of single, double and sequential sampling plans, is then presented. The comparison is based on the probability of acceptance in the presence of inspection errors. The three plans are designed for the same (AQL, α), (RQL, β) requirements, for the purpose of comparison.

The Chapter V deals with the variable sampling plans in the presence of the measurement errors. In this chapter, the effect of measurement errors on variable acceptance sampling is analysed. A compensating design procedure is also discussed. The case of known standard deviation is considered. A sequential probability ratio based plan is discussed which tests the mean of normal population in the presence of measurement errors. The case considered is of a single sided alternative with known standard deviation.

The Chapter VI is the concluding chapter which summarizes the results and developes logical interpretations. It also gives various interesting directions, one may work on for further research.

The Bibliography is provided to give a list of references in the end.

CHAPTER II

INSPECTION ERRORS AND SINGLE SAMPLING PLAN

2.1 Introduction:

Single sampling plan is probably the most extensively used plan amongst all acceptance plans. We would first present a brief description of a single sampling plan.

Generally, single sampling plans for attributes are characterized by two decision variables, denoted as,

n = The sample size

c = The acceptance number

The plan proceeds as follows. From a lot, of size N, a random sample of size n is drawn and each item of sample is inspected and classified as good or defective. If the number of defectives found in the sample exceeds the acceptance number c, the lot is rejected, otherwise it is accepted.

In this chapter, we will consider the effect of inspection errors on the various measures of the single sampling plan, such as probability of acceptance, average outgoing quality etc. Following that we will analyze certain aspects of design of this plan when inspection errors are present.

The single sampling plan is usually designed on the basis of quality protection requirements, i.e. to satisfy certain (AQL, α), (RQL, β) requirements. This, however, does not directly take any economic aspect into consideration. The plan is also designed on the basis of average outgoing quality (AOQL) and minimum average total inspection (ATI). This design can be termed as semi-economic design. Recently, there has been an increasing trend of designing a sampling plan based on the economics taking various costs, such as cost of inspection, cost of repair etc, into consideration.

We will see in following sections, how the above three procedures can be modified in presence of errors so as the requirements of (1) quality protection, (2) AOQL and ATI and (3) minimum expected cost are satisfied.

2.2 Effect of Errors on Some Performance Measures:

In case of inspection for acceptance sampling for attributes, two kinds of inspection errors are possible. An item which is good may be erroneously classified as defective (Type I error) or an item which is defective may be erroneously classified as good (type II error). At this stage, let us introduce following notations:

E_1 - The event that a good item is classified as defective.

E_2 - The event that a defective item is classified as good.

A - The event that an item is defective.

B - The event that an item is classified as defective.

Furthermore, let,

e_1 = Prob $\{E_1\}$ = The probability that E_1 occurs.

e_2 = Prob $\{E_2\}$ = The probability that E_2 occurs.

2.2.1 Apparent Incoming Quality:

Incoming quality cannot be termed as a performance measure, but all other performance measures are functions of incoming quality. So we start with the effect of errors on the incoming quality.

The erroneous inspector while making type I and/or type II errors does not recognize the incoming quality as such. Instead, he behaves as a perfect inspection would, to some other incoming quality, which we can term as apparent incoming quality. Then, if p is the true incoming quality (fraction defective) it can be written as,

$$p = \text{Prob}\ \{A\} = \text{Probability that event A occurs.}$$

Then apparent incoming quality denoted by p_e will be given by,

$$p_e = P\{B\} = \text{Probability that event B occurs.}$$

$$p_e = P(A)\ .\ P(E_2) + P(\bar{A})\ .\ P(E_1)$$

$$p_e = p\ (1 - e_2) + (1 - p)\ e_1 \qquad (2.1)$$

The first term on right hand side of Eq. (2.1) gives the proportion of actual defectives that will be called as defectives, and the second term gives the proportion of good items that will be classified as defectives. The above relation gives apparent incoming quality in terms of true incoming quality, (or true fraction defective) and inspection performance errors (e_1, e_2).

At this stage it will be useful to have an analysis for the distribution of observed defectives. We know that type B OC curve in the attribute sampling plans is simply the probability of acceptance versus process fraction defective. The Binomial mass function is often used for assessing the probability of acceptance [Duncan, (5)]. We would like to know what mass function would be appropriate for marginal distribution of the number of observed defectives in a sample [Collins et al (17)].

<u>Conditional Distribution of Observed Defectives:</u>

We would develop in this section the conditional distribution of the observed number of defectives, x_e, in the sample, which depends upon the prior distribution of actual number of defectives present in the sample. Let i denote the number of actual defectives correctly classified then the distribution of i is described by binomial mass function because the item which is defective may be classified as defective or good in presence of inspection error (type II error).

It means classifying a defective as good or as defective also forms a Bernoullies trial. The mass function $g_x(i)$ describing the distribution of actual defectives (out of x) classified correctly (i) is given by,

$$g_x(i) = \binom{x}{i} (1 - e_2)^i \, e_2^{\,x-i} \qquad (2.2)$$

The number of good items in the sample is, therefore, n - x. The number of good items that will be erroneously classified as defectives is x_e-i, where x_e is total number of observed defectives and i is number of actual defectives classified correctly. The mass function g_{n-x} (x_e-i) gives distribution of good items erroneously classified as defectives (type I error). The mass function $g_{n-x}(x_e-i)$ is given by,

$$g_{n-x}\,(x_e-i) = \binom{n-x}{x_e-i} \, e_1^{\,x_e-i} (1 - e_1)^{n-x-x_e+i} \qquad (2.3)$$

Then the conditional probability distribution $q_n(x_e/x)$ of having x_e as observed number of defectives given that x is actual number of defectives is given by,

$$q_n(x_e/x) = \sum_{i=\text{Max}\,[x_e-(n-x),0]}^{\text{Min}\,(x,x_e)} \binom{x}{i}(1-e_2)^i \, e_2^{\,x-i}\binom{n-x}{x_e-i} \, e_1^{\,x_e-i} (1-e_1)^{n-x-x_e+i} \qquad (2.4)$$

<u>Marginal Distribution of Observed Defectives:</u>

Once we have obtained conditional distribution of the observed defectives $q_n(x_e/x)$, we can get the marginal

distribution $g_n(x_e)$ of the observed number of defectives. The marginal or unconditional distribution $g_n(x_e)$ is given by [Collins et al (17)],

$$g_n(x_e) = \sum_{x=0}^{n} q_n(x_e/x) \cdot g_n(x)$$

$$g_n(x_e) = \sum_{x=0}^{n} \sum_{i=\mathrm{Max}[x_e-(n-x),0]}^{\mathrm{Min}(x,x_e)} \binom{x}{i} (1-e_2)^i \, e_2^{x-i}$$

$$\cdot \binom{n-x}{x_e-i} e_1^{x_e-i} (1-e_1)^{n-x-x_e+i} \binom{n}{x} p^x (1-p)^{n-x}$$

where $g_n(x) = \binom{n}{x} p^x (1-p)^{n-x}$.

$$g_n(x_e) = \binom{n}{x_e} \sum_{i=0}^{x_e} \binom{x_e}{i} [p(1-e_2)]^i [(1-p)e_1]^{x_e-i}$$

$$\cdot \sum_{x-i=0}^{n-x_e} \binom{n-x}{x_e-i} (pe_2)^{x-i} [(1-p)(1-e_2)]^{n-x-x_e+i}$$

which can be simplified to the following form by substituting for the binomial summations,

$$g_n(x_e) = \binom{n}{x_e} [p(1-e_2)+(1-p)e_1]^{x_e} [pe_2+(1-p)(1-e_1)]^{n-x_e} \tag{2.5}$$

The equation (2.5) can be simplified by substituting the value of p_e from equation (2.1),

$$g_n(x_e) = \binom{n}{x_e} p_e^{x_e} (1-p_e)^{n-x_e} \tag{2.6}$$

The equation (2.6) gives the marginal distribution of the number of observed defectives. The relation also indicates that modified Binomial (n, p_e) is the distribution which can be used in the expression for the OC curve in presence of inspection errors.

2.2.2 Probability of Acceptance:

It has been shown by Hald (17) that a Binomial distribution is applicable to hypergeometric sampling i.e. if a lot of size N is formed from a Binomial process, and a sample of n items is drawn from such a lot, then the number of defectives in the sample is described by a Binomial mass function. Probability of acceptance Pa in absence of errors, for a given incoming quality p is given by,

$$Pa = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \tag{2.7}$$

$$Pa = \sum_{x=0}^{c} g_n(x)$$

where $g_n(x)$ is the marginal distribution of the number of defectives x, in a sample of size n.

Let us define apparent probability of acceptance in presence of errors as Pa_e. From Eq. (2.6) Pa_e is given by,

$$Pa_e = \sum_{x_e=0}^{c} \binom{n}{x_e} p_e^{x_e} (1-p_e)^{n-x_e} \tag{2.8}$$

where x_e is the observed number of defectives. We note that

above expression for Pa_e is similar to Pa where x is replaced by x_e and p by p_e.

To illustrate the effect of inspection error on probability of acceptance we consider a typical single sampling plan ($N = 1000$, $n = 65$, $c = 3$). This plan was found for error-free inspection for ($AQL = 0.02$, $\alpha = 0.05$) and ($RQL = 0.12$, $\beta = 0.05$) requirements. For this plan, a few representative incoming quality levels were considered and the effect of few selected error pairs (e_1, e_2) on probability of acceptance was studied. The results are shown in Tables 1 and 2. Table 1, shows probabilities of acceptance, Table 2 shows, the percentage change in probability of acceptance from that with error-free inspection.

Now for the plan ($N = 1000$, $n = 65$, $c = 2$) found for the error free inspection to satisfy ($AQL = 0.01$, $\alpha = 0.05$), ($RQL = .08$, $\beta = 0.10$) requirements was considered. For this plan and few of the above error pairs OC curves were plotted and are shown in the Fig. 1. The figure shows that, when we are using the plan with an erroneous inspection who is commiting type II errors i.e. $e_1 = 0.0$, $e_2 > 0$, the probability of acceptance increases for all incoming quality levels. The same results are shown in Table 1. This is explainable. The inspector is erroneously classifying defectives as good and hence accepting the lot more often. At quality level $p = 0$, i.e. perfect incoming quality, there will be no defective

TABLE 1: PROBABILITY OF ACCEPTENCE IN PRESENCE OF ERRORS(SINGLE SAMPLING PLAN)

PLAN: (N=1000, n=65, c=3)

(AQL=0.02, α=0.05, RQL=0.12, β=0.05)

	ERROR-PAIRS	INCOMING QUALITY (p)					
	(e1,e2)	0.01	0.03	0.05	0.07	0.09	0.11
1	(0.000,0.000)	0.9958	0.8689	0.5900	0.3246	0.1524	0.0633
2	(0.000,0.050)	0.9965	0.8855	0.6272	0.3650	0.1828	0.0814
3	(0.000,0.075)	0.9968	0.8934	0.6458	0.3863	0.2178	0.0921
4	(0.000,0.100)	0.9971	0.9011	0.6645	0.4083	0.2178	0.1040
5	(0.000,0.125)	0.9974	0.9085	0.6830	0.4310	0.2372	0.1172
6	(0.000,0.150)	0.9977	0.9156	0.7015	0.4544	0.2578	0.1317
7	(0.000,0.200)	0.9981	0.9289	0.7378	0.5029	0.3029	0.1654
8	(0.010,0.000)	0.9572	0.7421	0.4544	0.2323	0.1036	0.0414
9	(0.025,0.000)	0.8104	0.5278	0.2847	0.1331	0.0556	0.0212
10	(0.050,0.000)	0.4544	0.2394	0.1110	0.0465	0.0179	0.0064
11	(0.075,0.000)	0.1920	0.0881	0.0369	0.0143	0.0052	0.0018
12	(0.100,0.000)	0.0663	0.0277	0.0108	0.0039	0.0014	0.0004
13	(0.010,0.100)	0.9652	0.7842	0.5242	0.2997	0.1518	0.0697
14	(0.050,0.100)	0.4680	0.2674	0.1374	0.0648	0.0284	0.0117
15	(0.025,0.200)	0.8360	0.6160	0.3988	0.2328	0.1249	0.0625
16	(0.050,0.200)	0.4818	0.2976	0.1687	0.0891	0.0443	0.0209
17	(0.100,0.100)	0.0694	0.0322	0.0141	0.0059	0.0023	0.0009
18	(0.125,0.125)	0.0209	0.0094	0.0041	0.0017	0.0007	0.0003

TABLE 2: PERCENTAGE CHANGE IN PROBABILITY OF ACCEPTENCE FOR VARIOUS ERROR-PAIRS(SINGLE SAMPLING PLAN)

PLAN: (N=1000,n=65,c=3)

(AQL=0.02,α=0.05,RQL=0.12,β=0.05)

	ERROR-PAIRS	INCOMING QUALITY (p)					
	(e1,e2)	0.01	0.03	0.05	0.07	0.09	0.11
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	0.07	1.91	6.31	12.45	19.95	28.59
3	(0.000,0.075)	0.10	2.82	9.46	19.01	31.04	45.50
4	(0.000,0.100)	0.13	3.71	12.63	25.79	42.91	64.30
5	(0.000,0.125)	0.16	4.56	15.76	32.78	55.64	85.15
6	(0.000,0.150)	0.19	5.37	18.90	39.99	69.16	108.06
7	(0.000,0.200)	0.23	6.91	25.05	54.93	98.75	161.30
8	(0.010,0.000)	-3.68	-14.59	-22.98	-28.43	-32.02	-34.60
9	(0.025,0.000)	-18.62	-39.26	-51.75	-59.00	-63.52	-66.51
10	(0.050,0.000)	-54.37	-72.45	-81.19	-85.67	-88.25	-89.89
11	(0.075,0.000)	-80.72	-89.86	-93.75	-95.59	-96.59	-97.16
12	(0.100,0.000)	-93.34	-96.81	-98.17	-98.80	-99.08	-99.37
13	(0.010,0.100)	-3.07	-9.75	-11.15	-7.67	-0.39	10.11
14	(0.050,0.100)	-53.00	-69.23	-76.71	-80.04	-81.36	-81.52
15	(0.025,0.200)	-16.05	-29.11	-32.41	-26.28	-18.04	-1.26
16	(0.050,0.200)	-51.62	-65.75	-71.41	-72.55	-70.93	-66.98
17	(0.100,0.100)	-93.03	-96.29	-97.61	-98.18	-98.49	-98.58
18	(0.125,0.125)	-97.90	-98.92	-99.31	-99.48	-99.54	-99.53

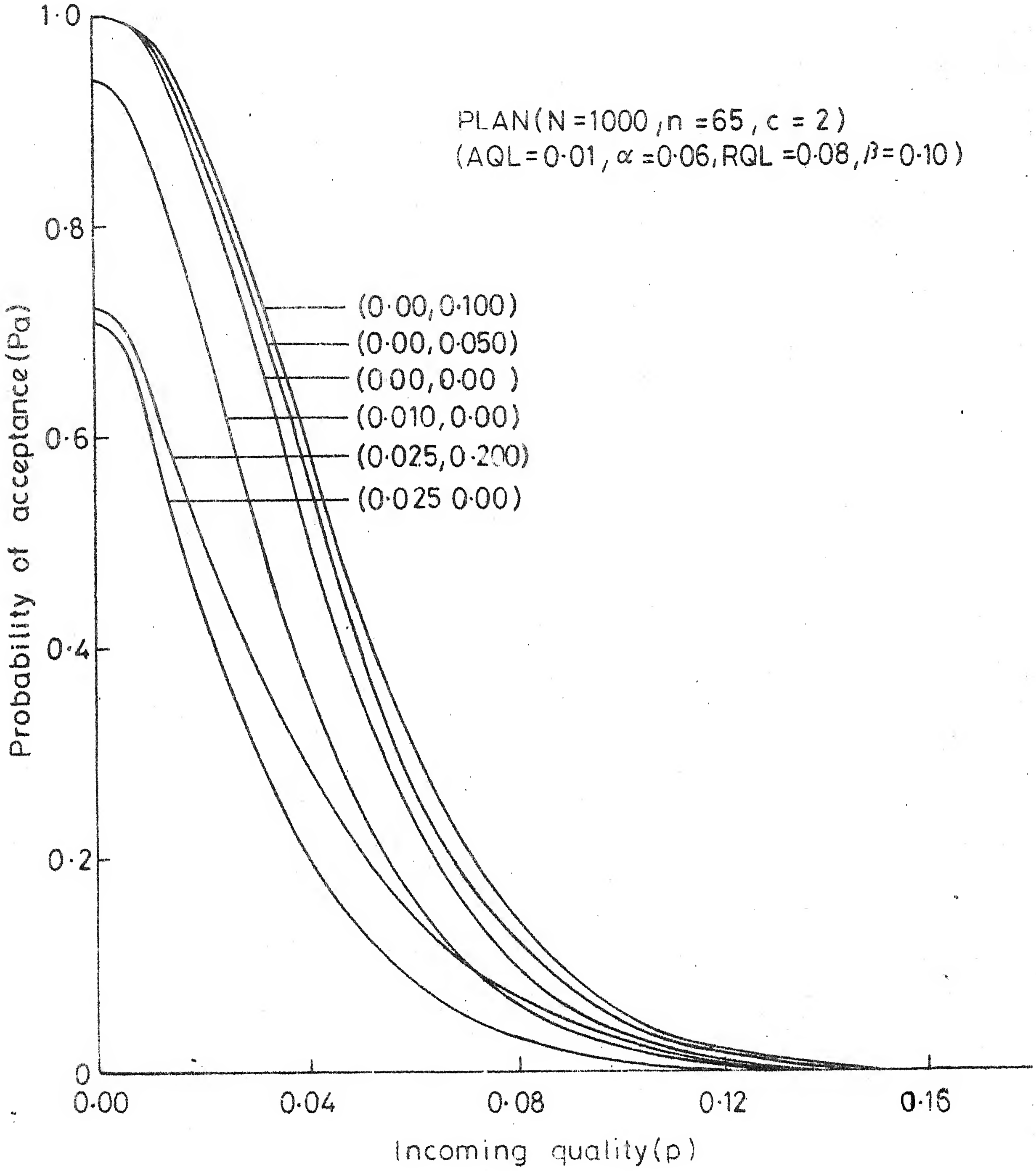

Fig. 1 OC Curve in presence of inspection errors
(Single sampling plan)

present and hence $Pa_e = Pa = 1$.

When the inspector is making type I errors, i.e., he is classifying good items as defectives($e_1 > 0$, $e_2 = 0$), he will be rejecting more often and hence Pa_e will be less than Pa for all incoming quality levels. This is shown both in Fig. 1 and in Table 1. The probability of acceptance at p = 0, is 1 in the absence of errors. But P_{ae} will be less than 1 at p = 0, because few of good item will be classified incorrectly. Pa_e would never be 1 in presence of type I errors.

Table 2 shows the percentage change in value of Pa_e from Pa, i.e. $\frac{Pa_e - Pa}{Pa} \times 100$. It can be seen that the effect of type I error is more significant compared to that of type II error. As type I error is incorrect classification of good items, and usually we deal with low values of p (say $p \leq 0.20$) i.e. there are much more good items than defective ones, hence type I error effects are more significant. The effect of type II error increases with increase in value of p, due to more number of defectives being present. Same is the case with type I error also, but because of low value of Pa at higher values of p, the percentage change increases as shown in Table 2.

2.2.3 Average Outgoing Quality (AOQ):

The performance measure average outgoing quality (AOQ) is a useful measure when we have rectifying inspection. It is defined as,

$$AOQ = \frac{\text{Expected number of defectives remaining in the accepted lot after inspection}}{\text{Total number of items in the accepted lot}}$$

If we assume perfect inspection with replacement of defective items, and rejected lots being put to 100 percent inspection, AOQ is given by [Duncan (5)],

$$AOQ = \frac{(N-n)p\,Pa}{N} \qquad (2.9)$$

If we assume perfect inspection and defectives found being scrapped and not replaced, then AOQ is given by [Duncan (5)],

$$AOQ = \frac{(N-n)\,p\,Pa}{N - np - (1-Pa)\,(N-n)\,p} \qquad (2.10)$$

Average outgoing quality expression is to be modified to account for the inspection errors. In case of perfect inspection we had an implicit assumption that replacement items are not inspected, i.e. they are taken from a good lot. In case of erroneous inspection, we will however, relax this assumption and inspect the replacement items also.

If n items have been sampled np will be the expected number of defectives present and np_e will be expected number of defectives observed. These np_e items have to be replaced,

by good items after inspection. Firstly n items will be inspected, followed by inspection of np_e replacement items, then np_e^2 items will be inspected, and so on. Thus total number of items inspected for sample and for replacement items to replenish the sample is given by,

$$n + np_e + np_e^2 + \ldots = \frac{n}{1-p_e} \tag{2.11}$$

If $\frac{n}{1-p_e}$ is the effective number of items inspected, the expected number of defectives which will erroneously be classified as good and accepted, while inspecting this effective sample can be given by,

$$\frac{n}{1-p_e} \; pe_2 \tag{2.12}$$

Similarily the rejected lot will be subjected to 100 percent inspection and hence N - n items will be inspected first, followed by the inspection of replacement items required to replenish the rejected portion of the lot. Hence, expected number of defectives which will be classified as good in the inspection of rejected portion of lots and replacement items thereafter to replenish the rejected portion of lot, is given by,

$$\frac{N - n}{1 - p_e} \; pe_2 \; (1-Pa_e) \tag{2.13}$$

Expected number of defectives in the accepted portion of the lot is given by,

$$(N - n) \; p \; Pa_e \tag{2.14}$$

Hence we can get the expected total number of defectives in the finally accepted lot by adding the above three terms. The expected total number of defectives is given by,

$$\frac{n\ pe_2}{1-p_e} + p(N-n)\ Pa_e + \frac{p(N-n)(1-Pa_e)\ e_2}{1-p_e} \qquad (2.15)$$

Then AOQ_e with replacement in presence of errors is given by,

$$AOQ_e = \left[\frac{npe_2}{1-p_e} + p(N-n)\ Pa_e + p\ \frac{(N-n)(1-Pa_e)e_2}{1-p_e}\right]/N$$

$$= \frac{npe_2 + p(N-n)\ Pa_e\ (1-p_e) + p(N-n)(1-Pa_e)e_2}{N(1-p_e)} \qquad (2.16)$$

AOQ_e without replacement in presence of errors is given by,

$$AOQ_e = \frac{npe_2 + p(N-n)\ Pa_e + p(N-n)(1-Pa_e)e_2}{N-np_e - (N-n)(1-Pa_e)\ p_e} \qquad (2.17)$$

The above equation (2.17) is for the case when rectifying inspection is done without replacement. In such a case no defective is introduced through replacement and there is no replenishment of lot size.

The effect of inspection errors is illustrated in Tables 3 and 4, taking the same plan as considered before ($N = 1000$, $n = 65$, $c = 3$). Table 3 shows the AOQ values with replacement in the absence and presence of errors. Table 4 shows percentage change in AOQ values in the presence of errors from that with error free inspection i.e.

$$\frac{AOQ_e - AOQ}{AOQ} \times 100$$

TABLE 3: AVERAGE OUTGOING QUALITY IN PRESENCE OF ERRORS
(SINGLE SAMPLING PLAN)

PLAN: (N=1000, n=65, c=3)

(AQL=0.02, α=0.05, RQL=0.12, β=0.05)

	ERROR-PAIRS (e1,e2)	INCOMING QUALITY (p) 0.01	0.03	0.05	0.07	0.09	0.11
1	(0.000,0.000)	0.14325	0.37495	0.42436	0.32683	0.19729	0.10012
2	(0.000,0.050)	0.14388	0.38621	0.46780	0.40556	0.29941	0.21617
3	(0.000,0.075)	0.14418	0.39141	0.48847	0.44412	0.35067	0.27490
4	(0.000,0.100)	0.14448	0.39362	0.50641	0.48220	0.40198	0.33414
5	(0.000,0.125)	0.14478	0.40096	0.52756	0.51965	0.45326	0.39384
6	(0.000,0.150)	0.14507	0.40534	0.54598	0.55633	0.50444	0.54398
7	(0.000,0.200)	0.14565	0.41334	0.58037	0.57722	0.60602	0.57538
8	(0.010,0.000)	0.13798	0.32026	0.37682	0.23394	0.13407	0.06550
9	(0.025,0.000)	0.11657	0.22778	0.20479	0.13105	0.07200	0.03350
10	(0.050,0.000)	0.06536	0.10331	0.07986	0.04582	0.02315	0.01010
11	(0.075,0.000)	0.02762	0.03803	0.02653	0.01439	0.00669	0.00278
12	(0.100,0.000)	0.00353	0.01195	0.00775	0.00397	0.00176	0.00070
13	(0.010,0.100)	0.14307	0.35119	0.41853	0.38535	0.37706	0.28768
14	(0.050,0.100)	0.07651	0.15282	0.17268	0.17632	0.19107	0.21392
15	(0.025,0.200)	0.12720	0.39696	0.38998	0.41739	0.43192	0.45712
16	(0.050,0.200)	0.08725	0.20026	0.26336	0.30971	0.35815	0.41560
17	(0.100,0.100)	0.02611	0.06499	0.09841	0.13291	0.16988	0.20964
18	(0.125,0.125)	0.02474	0.07114	0.11729	0.16510	0.21507	0.26727

TABLE 4: PERCENTAGE CHANGE IN AOQ FOR VARIOUS ERROR-PAIRS(SINGLE SAMPLING PLAN)

PLAN: (N=1000,n=65,c=3)

(AQL=0.02, =0.05,RQL=0.12, =0.05)

	ERROR-PAIRS (e1,e2)	INCOMING QUALITY (p) 0.01	0.03	0.05	0.07	0.09	0.11
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	0.44	3.00	10.74	24.07	51.76	115.91
3	(0.000,0.075)	0.65	4.39	15.11	35.89	77.74	174.57
4	(0.000,0.100)	0.86	4.98	19.81	47.54	103.75	233.74
5	(0.000,0.125)	1.07	6.94	24.32	58.99	129.74	293.37
6	(0.000,0.150)	1.27	8.11	28.65	70.22	155.68	443.33
7	(0.000,0.200)	1.68	10.24	36.75	91.91	207.17	476.90
8	(0.010,0.000)	-3.68	-14.59	-22.99	-28.42	-32.04	-34.58
9	(0.025,0.000)	-18.62	-39.25	-51.74	-58.98	-63.51	-66.54
10	(0.050,0.000)	-54.37	-72.45	-81.18	-85.67	-88.27	-89.91
11	(0.075,0.000)	-80.72	-89.86	-93.75	-95.60	-96.61	-97.22
12	(0.100,0.000)	-93.75	-96.81	-98.17	-98.79	-99.11	-99.30
13	(0.010,0.100)	-0.13	-6.34	-1.37	17.91	65.78	187.34
14	(0.050,0.100)	-46.59	-59.24	-59.31	-45.29	-3.15	113.66
15	(0.025,0.200)	-11.20	-18.13	-8.13	27.71	118.93	356.57
16	(0.050,0.200)	-39.09	-46.59	-37.94	-5.24	81.53	315.10
17	(0.100,0.100)	-81.77	-82.67	-76.81	-59.36	-13.89	109.39
18	(0.125,0.125)	-82.73	-81.03	-72.36	-49.48	9.01	166.95

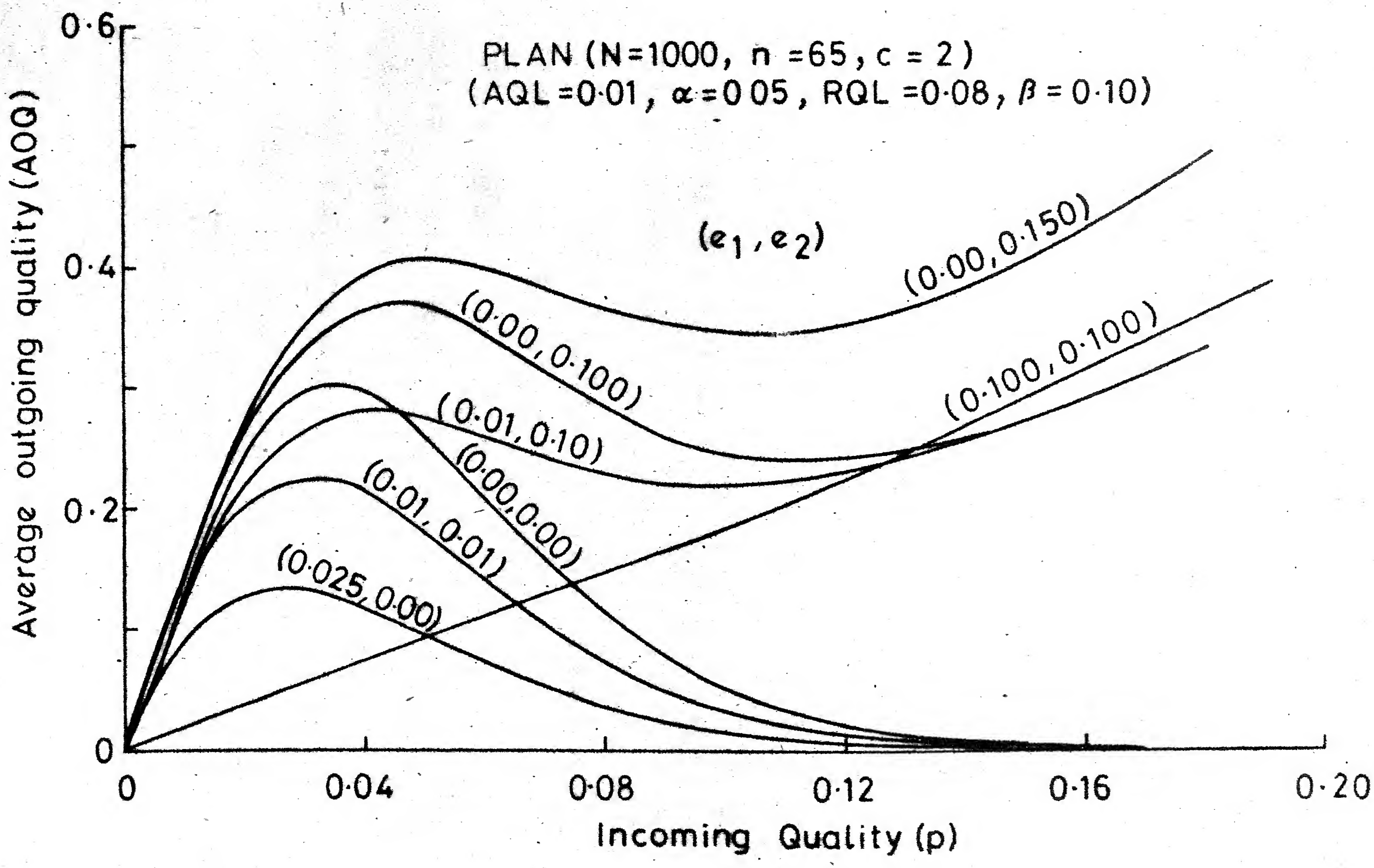

Fig. 2 Average outgoing quality in presence of inspection errors (Single sampling plan)

Figure 2, shows the variation of AOQ versus incoming quality in absence of, and in presence of inspection errors, for the plan (N = 1000, n = 65, c = 2). The curves shown are for the case when defective items are replaced. AOQ when defective items are replaced. AOQ curve is almost similar to one shown in case of no replacement. The values are, however, slightly different. The details can be had from [Case et al (23)].

Incorrect classification of good items would reduce the AOQ, due to the fact that more screening inspection will take place. The major portion of defectives are contributed by the accepted lots which have certain probability of acceptance. In presence of type I error ($e_1 > 0$, $e_2 = 0$) probability of acceptance reduces and hence better outgoing quality is resulted.

The incorrect classification of defective items has the effect of causing higher value of AOQ for all values of p. This is expected even intuitively because more defectives will now be present in the finally accepted lots than in the case of perfect inspection. This will happen because of erroneous classification of defective items as good.

There is another interesting point, to be noted in the case of type II errors. Near the point where Pa_e is very small, the AOQ curve rises due to increased number of defectives being classified as good, as p increases.

Furthermore as $p \to 1$, $AOQ \to 1$ as a limit. Therefore, for any given sampling plan encompassing type II errors, conventional concept of AOQ is not meaningful. In the Tables 3 and 4, and in Fig. 2, the results obtained are in accordance to explaination given above.

2.2.4 Average Total Inspection:

Average total inspection represents the long run average of total items inspected per lot.Totalinspection includes inspection required for replacement items for rectification. The average total inspection for the case when defectives found are replaced by good items and for the case when defectives found are not replaced and are just scrapped,in the presence of inspection errors is given by,

$$ATI_e = \frac{n + (1 - Pa_e)(N - n)}{1 - p_e} \quad \text{(with replacement)} \quad (2.18)$$

$$ATI_e = n + (1 - Pa_e)(N - n) \quad \text{(without replacement)} \quad (2.19)$$

The effect of inspection errors on average total inspection is illustrated in the Tables 5 and 6. Again the same sampling plan ($N = 1000$, $n = 65$, $c = 3$) is considered. The tables give the results for few representative values of incoming quality levels and several selected error pairs (e_1, e_2). Table 5 gives the ATI values for several error pairs at selected incoming quality levels. Table 6 gives the percentage change, i.e. $\frac{ATI_e - ATI}{ATI} \times 100$ in the value of ATI

TABLE 5: AVERAGE TOTAL INSPECTION IN PRESENCE OF ERRORS(SINGLE SAMPLING PLAN)

PLAN: (N=1000,n=65,c=3)

(AQL=0.02,α=0.05,RQL=0.12,β=0.05)

	ERROR-PAIRS (e1,e2)	INCOMING QUALITY 'p' 0.01	0.03	0.05	0.07	0.09	0.11
1	(0.000,0.000)	69.59	193.40	471.94	748.94	942.32	1057.12
2	(0.000,0.050)	68.91	177.11	434.20	705.67	906.61	1031.65
3	(0.000,0.075)	68.59	169.35	415.35	683.04	887.13	1017.37
4	(0.000,0.100)	68.30	161.85	396.57	659.78	866.52	1001.94
5	(0.000,0.125)	68.02	154.63	377.91	635.93	844.76	985.27
6	(0.000,0.150)	67.76	147.69	359.41	611.52	821.83	967.77
7	(0.000,0.200)	67.30	134.67	323.05	561.26	772.41	926.97
8	(0.010,0.000)	105.20	318.76	611.52	850.17	1002.52	1091.02
9	(0.025,0.000)	251.04	535.53	792.11	965.57	1068.57	1129.59
10	(0.050,0.000)	511.52	842.27	993.00	1082.65	1137.40	1175.67
11	(0.075,0.000)	895.92	1022.69	1098.73	1146.92	1182.25	1212.70
12	(0.100,0.000)	1052.78	1115.82	1157.81	1190.34	1219.45	1247.92
13	(0.010,0.100)	99.39	276.94	539.23	775.85	943.06	1047.85
14	(0.050,0.100)	597.35	811.26	960.38	1054.96	1114.43	1154.78
15	(0.025,0.200)	225.74	445.52	669.83	849.70	975.62	1058.19
16	(0.050,0.200)	583.02	778.14	923.00	1021.37	1086.20	1130.21
17	(0.100,0.100)	1048.31	1107.84	1147.46	1178.33	1205.40	1230.51
18	(0.125,0.125)	1130.21	1162.76	1189.50	1213.90	1237.62	1261.53

TABLE 6: PERCENTAGE CHANGE IN ATI FOR VARIOUS ERROR-PAIRS(SINGLE SAMPLING PLAN)

PLAN: (N=1000,n=65,c=3)

(AQL=0.02,α=0.05,RQL=0.12,β=0.05)

	ERROR-PAIRS	INCOMING QUALITY (p)					
	(e1,e2)	0.01	0.03	0.05	0.07	0.09	0.11
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	-0.98	-8.42	-8.00	-5.76	-3.79	-2.41
3	(0.000,0.075)	-1.44	-12.44	-11.99	-8.89	-5.85	-3.76
4	(0.000,0.100)	-1.85	-16.31	-15.97	-11.90	-8.04	-5.22
5	(0.000,0.125)	-2.26	-20.05	-19.92	-15.09	-10.35	-6.80
6	(0.000,0.150)	-2.63	-23.63	-23.84	-19.35	-12.79	-8.50
7	(0.000,0.200)	-3.29	-30.37	-31.55	-25.06	-18.03	-12.31
8	(0.010,0.000)	51.17	64.82	29.58	13.52	6.39	3.21
9	(0.025,0.000)	260.74	176.90	67.86	28.92	13.39	6.86
10	(0.050,0.000)	778.75	335.51	110.41	44.56	20.70	11.21
11	(0.075,0.000)	1187.43	428.80	132.81	53.14	25.46	14.72
12	(0.100,0.000)	1412.83	476.95	145.33	59.94	29.41	18.05
13	(0.010,0.100)	42.82	43.20	14.26	3.59	0.08	-0.88
14	(0.050,0.100)	758.38	319.47	103.50	40.85	18.26	9.24
15	(0.025,0.200)	224.39	130.36	41.93	13.45	3.53	0.10
16	(0.050,0.200)	737.79	302.35	95.58	36.38	15.27	6.91
17	(0.100,0.100)	1406.41	472.82	143.14	57.33	27.89	16.40
18	(0.125,0.125)	1524.10	501.22	152.04	62.08	31.34	19.34

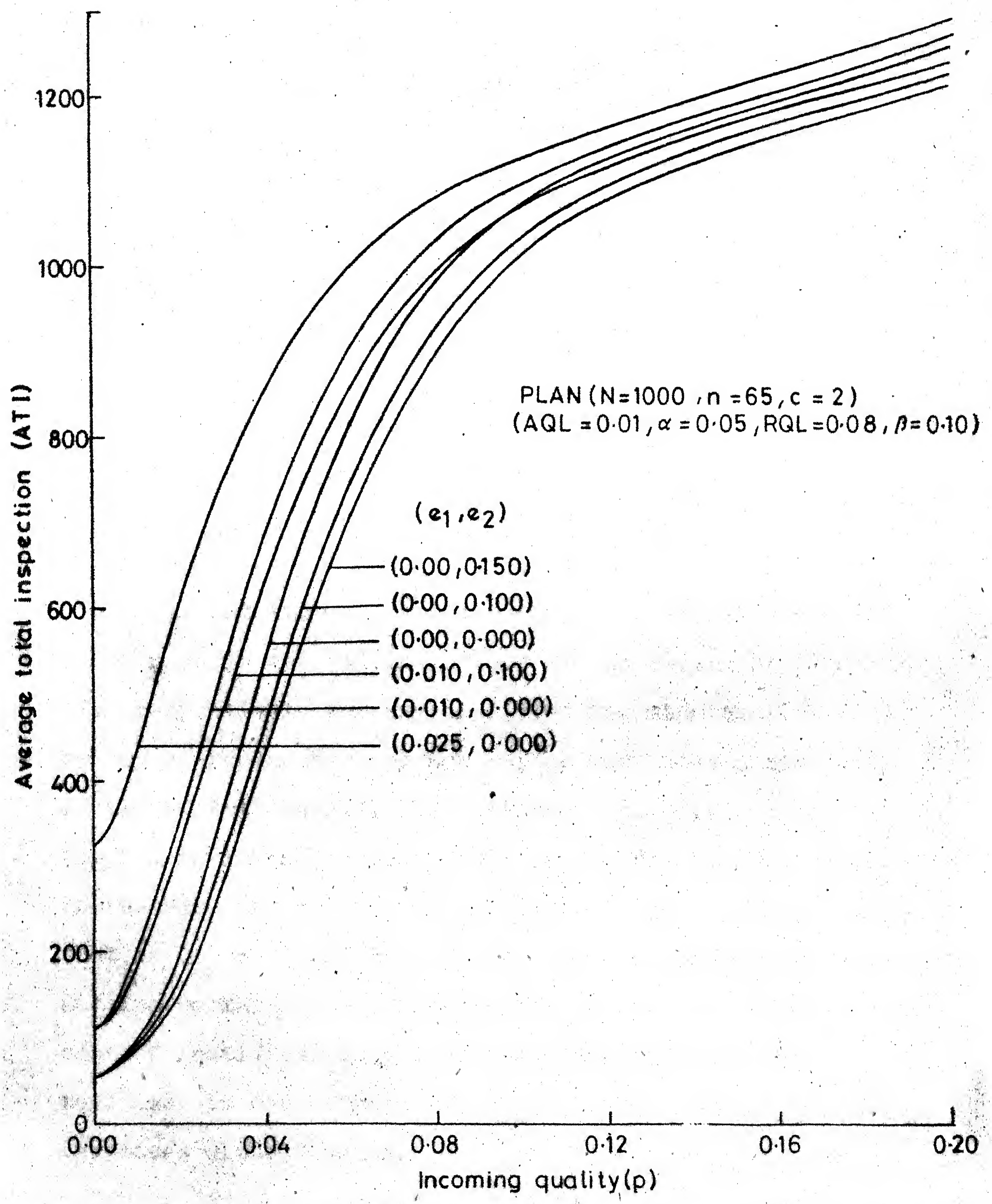

Fig. 3 Average total inspection in presence of inspection errors (Single sampling plan)

in the presence of errors from that in case of error-free inspection. Figure 3 shows variation of ATI values versus incoming quality levels for few of the error pairs, for the plan (N = 1000, n = 65, c = 2).

It can be seen from the tables 5 and 6 that, the effect of type I error is much more drastic as compared to that of type II errors. The ATI values given in Tables, and used for computing ATI curves are for the case when defectives found are replaced. It can be seen that ATI is frequently in excess of lot size both in the presence and in absence of inspection errors for high value of p.

As intuitively expected, type I error and type II error have in general the effect of increasing or decreasing ATI respectively for any specified incoming quality. The policy of replacement or non replacement does not significantly affect the ATI. The distinguishing difference amongst the two is in the values of ATI as the fraction defective increases. Under a non replacement policy, it can be seen that as $Pa_e \rightarrow 0$, $ATI_e \rightarrow N$. Thus, for large fraction defectives and for normal range of errors ATI_e equals lot size. In the case of rectification policy, when defectives found are replaced, it can be seen when $P_e \rightarrow 1$, $Pa_e \rightarrow 0$ and that ATI_e increases without bound.

It may be recalled at this stage, that, for the case of perfect inspection, with replacement of all defectives ATI is

given by, [Duncan (5)],

$$ATI = n + (N - n)\ (1 - Pa) \tag{2.20}$$

In the above expression, it is implicitly assumed that replenishment items are not being inspected i.e. they are being taken from a lot of good items. If the replacement items required to replenish lot are inspected ATI expression will be modified as,

$$ATI' = \frac{n + (N - n)\ (1 - Pa)}{1 - p} \tag{2.21}$$

If ATI values are computed by above equation for a specific plan, we will see that ATI exceeds lot size at higher values of p (even in the absence of errors).

2.3 Analysis and Design of Single Sampling Plan Based Upon (AQL, α), (RQL, β) Requirements:

Analysis:

Each single sampling plan has, associated with it, an operating characteristics curve. The design of single sampling plan is often based on two points on the theoretical O.C. curve. These points may be denoted as Acceptable quality level, AQL and reject quality level RQL (also known as lot tolerance proportion defective (LTPD). These are associated with the risk probabilities known as producers risk α and consumers risk β respectively.

AQL = $p_{1-\alpha}$ = The actual fraction defective considered to be an acceptable quality level and at which it is desirable to accept $(1-\alpha)$ fraction of such lots.

RQL (LTPD) = p_β = The actual fraction defective considered to be a lot tolerance limit (Reject quality level) and at which it is desirable to have a β probability of accepting such a lot.

Let us consider the points $p_{1-\alpha}$, p_β and the development of a sampling plan assuming perfect inspection. If this plan is used with erroneous inspection the OC curve will shift as discussed in section 2.2.2. We may like to shift the OC curve up, in presence of type I error and down, in presence of type II errors so as two points still fall on the desired OC curve. In otherwords, we may force the observed OC curve (with imperfect inspector) to fit those two points.

In order to actually attain levels $p_{1-\alpha}$ and p_β, it is only necessary to design the sampling plan based on $p_{e,1-\alpha}$ and $p_{e,\beta}$, which are given by,

$$\begin{aligned} AQL_e &= p_{e,1-\alpha} = p_{1-\alpha}(1-e_2) + (1-p_{1-\alpha})\, e_1 \\ RQL_e &= p_{e,\beta} = p_\beta(1-e_2) + (1-p_\beta)\, e_1 \end{aligned} \qquad (2.22)$$

If the observed OC curve fits $p_{e,1-\alpha}$ and $p_{e,\beta}$, then the actual OC curve fits the points $p_{1-\alpha}$ and p_β. The point to be

noted is that only two points are being forced to fit the desired curve, the rest of the observed OC curve will be still above the required curve (in case of type I error) or will be below the desired curve (in presence of type II error) or may cross the curve in between AQL and RQL, if both errors are present.

Design of Plan:

A single sampling plan design based on (AQL,α), (RQL,β) requirement is done usually by explicit enumeration. The two decision variables required to be found are sample size, n and acceptance number c. The two points given above, give two relations which have to be satisfied. The relations to be satisfied are given by, (in absence of errors),

$$P_a\ (AQL) = 1 - \alpha$$

and $$P_a\ (RQL) = \beta$$

Let us represent $p_{1-\alpha}$ by p_1 and p_β by p_2 for simplicity. Then the above relations can be expressed mathematically as,

$$\sum_{x=0}^{c} \binom{n}{x} p_1^{\,x} (1-p_1)^{n-x} = 1-\alpha$$

(2.23)

$$\sum_{x=0}^{c} \binom{n}{x} p_2^{\,x} (1-p_2)^{n-x} = \beta$$

The above equations can be solved only by explicit enumeration, noting that n and c are integers. The above equations can be modified to account for inspection errors as follows,

$$\sum_{x_e=0}^{c} \binom{n}{x_e} p_{1e}^{\,x_e} (1-p_{1e})^{n-x_e} = 1-\alpha$$

$$\sum_{x_e=0}^{c} \binom{n}{x_e} p_{2e}^{\,x_e} (1-p_{2e})^{n-x_e} = \beta \qquad (2.24)$$

where, $p_{ie} = p_i\,(1-e_2) + (1-p_i)\,e_1$

The procedure adopted is based on Guenthers procedure. The procedure starts with minimum possible value of acceptance number c, i.e. zero. It increases n by one at each iteration until probability of acceptance at RQL falls below β. For this set of c and n, it checks for AQL and α requirements. The probability of acceptance at AQL may fall below $1 - \alpha$ and in such a case c is increased by one and whole step is repeated. This procedure works systematically in direction of explicit enumeration and insures minimum n and c.

The procedure was originally used for poisson probability distribution. For the design of single sampling plan in presence of errors we have considered the Binomial probability distribution. It was coded in form of a computer program and results obtained are given in Table 7. Single sampling plans for various sets of (AQL, α), (RQL, β) requirements and several error pairs (e_1, e_2) were determined. These sampling plans have infact compensated for the presence of inspection errors. The Table 7 can be used to study the effect of inspection errors on sample size and acceptance number.

TABLE 7: SINGLE SAMPLING PLANS IN PRESENCE OF ERRORS FOR VARIOUS VALUES OF AQL,RQL

$\alpha=0.05;\beta=0.10$ FOR ALL AQL,RQL REQUIREMENTS

	ERROR-PAIRS	AQL/RQL						
	(e1,e2)	0.01/0.08	0.01/0.12	0.01/0.10	0.01/0.16	0.015/0.12	0.02/0.12	0.025/0.16
1	(0.000,0.000)	(65,2)	(31,1)	(52,2)	(23,1)	(43,2)	(54,3)	(32,2)
2	(0.000,0.050)	(69,2)	(33,1)	(55,2)	(25,1)	(45,2)	(57,3)	(34,2)
3	(0.000,0.075)	(71,2)	(34,1)	(56,2)	(25,1)	(47,2)	(59,3)	(35,2)
4	(0.000,0.100)	(73,2)	(35,1)	(58,2)	(26,1)	(48,2)	(60,3)	(36,2)
5	(0.000,0.125)	(75,2)	(36,1)	(60,2)	(27,1)	(49,2)	(62,3)	(37,2)
6	(0.000,0.150)	(77,2)	(37,1)	(61,2)	(28,1)	(51,2)	(64,3)	(38,2)
7	(0.000,0.200)	(82,2)	(40,1)	(65,2)	(29,1)	(54,2)	(68,3)	(40,2)
8	(0.010,0.000)	(88,4)	(40,2)	(60,3)	(30,2)	(50,3)	(61,4)	(38,3)
9	(0.025,0.000)	(112,7)	(55,4)	(74,5)	(36,3)	(64,5)	(72,6)	(50,5)
10	(0.050,0.000)	(157,14)	(77,8)	(104,10)	(44,5)	(64,9)	(99,11)	(62,8)
11	(0.075,0.000)	(209,24)	(93,12)	(131,16)	(56,8)	(106,14)	(118,16)	(72,11)
12	(0.100,0.000)	(257,36)	(116,18)	(169,25)	(70,12)	(127,20)	(143,23)	(84,15)
13	(0.010,0.100)	(97,4)	(56,3)	(66,3)	(34,2)	(56,3)	(67,4)	(42,3)
14	(0.025,0.100)	(135,8)	(70,5)	(92,6)	(39,3)	(79,6)	(89,7)	(54,5)
15	(0.025,0.200)	(161,9)	(77,5)	(101,6)	(52,4)	(87,6)	(98,7)	(69,6)
16	(0.050,0.200)	(232,19)	(108,10)	(149,13)	(67,7)	(125,12)	(141,14)	(88,10)
17	(0.100,0.100)	(323,44)	(141,21)	(203,29)	(86,14)	(158,24)	(175,27)	(106,18)
18	(0.125,0.125)	HIGH n,c	(179,31)	(259,43)	(103,19)	(200,35)	(220,39)	(130,25)

SINGLE SAMPLING PLAN(n,c)

In presence of type II errors ($e_1 = 0$, $e_2 > 0$), it can be seen that sample size increases for same c. That is, we will have to sample more and acceptance will be at the same c. In a way we are compensating for errors by allowing for defectives which are being erroneously classified as good (type II error). When we are accepting at acceptance number c, then infact we are accepting with actual number of defectives being less than or equal to $c + \delta$, δ being number of defectives that might have been erroneously classified as good. Thus sample size increases in presence of type II error, c does not change for practical range of quality, α and β.

In presence of type I error, ($e_1 > 0$, $e_2 = 0.0$) the whole plan gets modified. Both n and c are increased. But still the values of n and c show that in fact we are allowing for larger proportion of defectives for acceptance because some of good items will now be erroneously classified as defectives (type I error).

When both errors are present type I error is the dominating member because of already mentioned reasons. The plan gets modified with changed values of n and c.

At this point it is interesting to recall that the plan depends upon difference of p_1 and p_2 and risk probabilities α and β. Lesser the difference between p_1 and p_2 or lower the risk probabilities higher the sample size and the acceptance numbers. Let us try to analyze the effect of errors on this aspect.

Case I: when e_2 is present,

$$p_{1e} = p_1 (1 - e_2)$$

$$p_{2e} = p_2 (1 - e_2)$$

$$p_{2e} - p_{1e} = (p_2 - p_1)(1 - e_2)$$
$$< p_2 - p_1$$

Case II: when e_1 is present,

$$p_{1e} = p_1 + (1 - p_1)\, e_1$$

$$p_{2e} = p_2 + (1 - p_2)\, e_1$$

$$p_{2e} - p_{1e} = (p_2 - p_1) + (p_1 - p_2)\, e_1$$
$$= (p_2 - p_1)(1 - e_1)$$
$$< p_2 - p_1$$

Also one may check,

$$\frac{p_{2e}}{p_{1e}} < \frac{p_2}{p_1}$$

It can be seen both e_1 and e_2 reduce the difference between apparent AQL and RQL and hence sample size and acceptance number will increase. In case of e_1, the ratio RQL_e/AQL_e is less than RQL/AQL. It may lead to very high n and c which may require too large computational time or may be even infeasible if $n > N$. (It means even 100 percent inspection will not satisfy the requirements with such an erroneous inspector).

2.4 Analysis of Single Sampling Plan Based upon AOQL and ATI Requirements:

The design of single sampling plan based upon AOQL and ATI requirements can also be termed as semi-economic design. This tries, to have a plan which will not let the average quality of accepted lots to fall below certain specified limit (AOQL) and will require minimum average total inspection. (Thus, this plan tries to minimize the expected inspection cost).

Model Development Without Errors [Dodge and Romig (3)]:

The average outgoing quality AOQ, when inspection is error free is given by following expression,

$$AOQ = p\,\frac{N-I}{N} \qquad \text{(with replacement)} \tag{2.25}$$

where I = The average number of items inspected per lot.

AOQL is the maximum value of AOQ that will result under any incoming quality for any sampling plan. We are required to find that plan which will require minimum ATI at some specified incoming quality. Let value of p for which maximum value of AOQ occurs is designated as p_m, hence,

$$AOQL = p_m\,\frac{N-I}{N} \tag{2.26}$$

The value of p_m for which AOQ = AOQL can be determined by differentiating equation (2.25) with respect to p and equating to zero, then solving it for p,

$$\frac{dAOQ}{dp} = \frac{N-I}{N} - \frac{p}{N}\frac{dI}{dp} = 0 \tag{2.27}$$

For a single sampling plan I is given by,

$$I = n + (N - n)(1 - Pa) \tag{2.28}$$

For a Poisson distribution,

$$Pa = \sum_{x=0}^{c} \frac{e^{-pn}(pn)^x}{x!} \tag{2.29}$$

substituting value of P_a in Eq. (2.28), we get,

$$I = n + (N - n)\left(1 - \sum_{x=0}^{c} \frac{e^{-pn}(pn)^x}{x!}\right) \tag{2.30}$$

Substituting value of I in AOQ expression of Eq. (2.25), we get,

$$AOQ = \left[p(N - n) \sum_{x=0}^{c} \frac{e^{-pn}(pn)^x}{x!}\right]/N \tag{2.31}$$

differentiating equation (2.31) with respect to p, we get,

$$\frac{dAOQ}{dp} = \frac{N-n}{N}\left[\sum_{x=0}^{c} \frac{e^{-pn}(pn)^x}{x!} - \frac{e^{-pn}(pn)^{c+1}}{c!}\right] \tag{2.32}$$

By equating Eq. (2.32) to zero and solving for p, we will get the value of $p = p_m$, for which AOQ will be maximum (i.e. AOQL). Let,

$$p_m n = w \tag{2.33}$$

$$AOQL = \frac{N-n}{N} \frac{w}{n} \sum_{x=0}^{c} \frac{e^{-w} w^x}{x!}$$

$$= \frac{N-n}{Nn} y = y\left[\frac{1}{n} - \frac{1}{N}\right] \tag{2.34}$$

where,

$$y = w \sum_{x=0}^{c} \frac{e^{-w} w^x}{x!} \tag{2.35}$$

Solving Eq. (2.32) after equating to zero, we will get,

$$\sum_{x=0}^{c} \frac{e^{-w}\, w^{x}}{x!} - \frac{e^{-w}\, w^{c+1}}{c!} = 0$$

or
$$\sum_{x=0}^{c} \frac{e^{-w}\, w^{x}}{x!} = \frac{e^{-w}\, w^{c+1}}{c!} \qquad (2.36)$$

Substituting value of y from Eqn. (2.35), we get,

$$y = \frac{e^{-w}\, w^{c+2}}{c!} \qquad (2.37)$$

The above expression provides a basis for determining the value of y corresponding to a specific c. The value of w can be found by Newtons method of approximation on solving Eq. (2.36),corresponding y can be found by Eq. (2.37). Thus knowing y corresponding to a particular c we get value of n by Eq. (2.34). In this way for a selected value of c, we get n which will satisfy AOQL requirement. We can find ATI at specified p for this plan. The whole procedure can be repeated for c increased by one. ATI value now found can be compared with the one found before. This has to be repeated till we get a plan requiring minimum ATI [Dodge and Romig (3)].

The point to be noted in the above analysis that Poisson probability distribution has been used. The use of Binomial probability distribution will make the analysis more complex and we will need some heuristic to find a plan which will satisfy AOQL requirements and will require minimum ATI.

Analysis in the Presence of Errors:

We know that average outgoing quality (AOQ) and average total inspection (ATI) are modified in the presence of errors to following forms as given by Eqns. (2.16) and (2.18) respectively.

$$AOQ_e = \frac{npe_2 + p(N-n)\ (1-p_e)\ Pa_e + p(N-n)(1-Pa_e)e_2}{N(1-p_e)}$$

$$ATI = \frac{n + (1 - Pa_e)\ (N - n)}{1 - p_e}$$

Both the equations are for the case when defectives found are replaced, with replacement items required to replenish the lot, being put inspection. In such a case the variation of AOQ_e versus p was shown, for various error pairs in Table 2 and Figure 3. For the plan considered before (N = 1000 n = 65, c = 3) and for error pairs (0.050, 0.100) and (0.025, 0.200), the AOQ_e showed pattern as shown in the following figure 4.

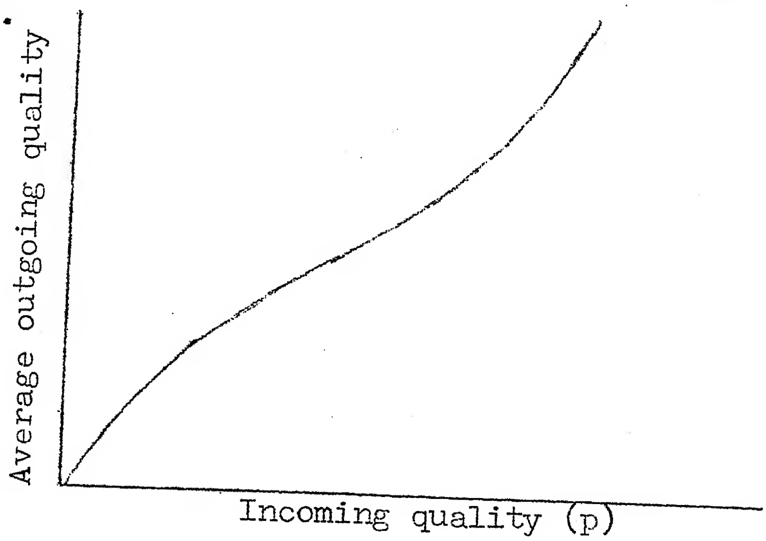

Fig. 4: AOQ curve showing non-decreasing trends.

The above set of error pairs have type I error as 5 percent and 2.5 percent respectively. These values are not uncommon. For these error pairs, AOQ does not show any decline in the whole range of incoming quality. So the concept of AOQL based design is not applicable here.

We can consider the cases when in presence of errors AOQ_e shows pattern as in Fig. 5. In such a case it may be worth while to find a plan which will satisfy the requirements of AOQL and ATI and the corresponding point p_x. Point p_x indicates that selected plan cannot limit the fraction defective of outgoing lots if incoming quality exceeds p_x.

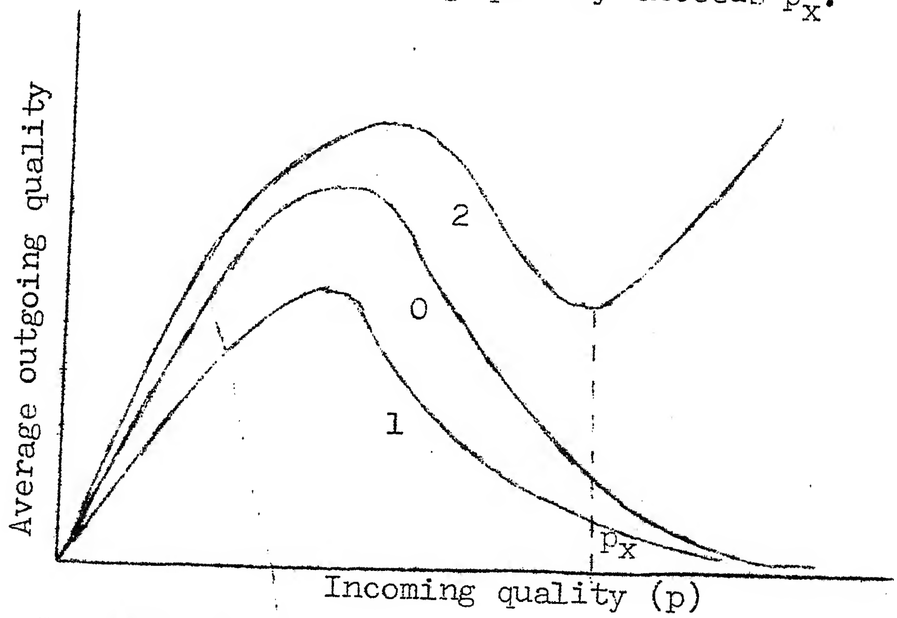

Fig. 5: AOQ in the presence of typical error pairs.

If only type I errors are present then pattern same as curve 1 will be observed. In such a case one can develope a heuristic starting from a plan based with error free assumption to satisfy AOQL and minimum ATI requirements even in the presence of errors.

If only type II errors are present or if both errors are present and AOQ_e shows a pattern as curve 2 in Fig. 5, then again a heuristic can be developed which will yield a plan satisfying AOQL and minimum ATI requirements, until incoming quality is better than p_x. Beyond p_x there is again no control on outgoing quality.

2.5 Economic Design of Single Sampling Plan:

The treatment so far dealt with sampling plans designed on purely statistical measures. The effect of inspection errors on economics of a sampling plan will now be considered. Since economic designs are receiving wide spread popularity, it seems natural to investigate the effect of (e_1, e_2) on economics of these plans. Following is the approach suggested by Case et al (27). In addition to the notations already mentioned following notations will be used.

x = Actual number of defectives in the sample of size n.

X = Actual number of defectives in the lot of size N.

u = $X-x$ = Number of defectives remaining in the uninspected portion of the lot

x_e = Apparent number of defectives in the n items sampled.

C_i = Cost of inspecting an item.

C_r = Cost of repair or rework of a defective item.

C_a = Cost attributed to a defective item in an accepted lot.

Worthram and Mogg (29) reported nine possible rectification policies, which can be considered in determining average

outgoing quality. Six of these possibilities were used by Worthram and Wilson (28) in designing an optimal sequential sampling plan. We will consider only one of these possibilities. The other possibilities will however, not alter the analysis much.

Sample and Lot Dispositions:

The following are three sample dispositions and the three lot dispositions. These two sets combine together giving nine rectification policies.

Sample:

(i) Discard the entire sample

(ii) Replace or repair all defectives found

(iii) Only enumerate defectives

Rejected Lots:

(i) Discard the entire lot

(ii) Inspect the rejected lot 100 percent and discard the defectives found

(iii) Inspect the rejected lots 100 percent and replace, repair or rework all the defectives found

We will consider (ii), (iii) policy for the sample and the rejected lots. We will have 100 percent inspection of rejected lots and repair, replace or rework all defectives found both, in the sample and in the rejected lots.

Model Formulation Without Errors:

For each of the above mentioned nine rectification policies, there are two courses of actions, either to accept the lot or to reject it. The action to accept the lot is taken when the number of defectives x found in the sample of size n, is such that $x \leq c$, otherwise action to reject the lot is taken. In the design we seek, the values of decision variables n and c, in such a way, that minimizes the expected total cost of quality control per lot. Let,

a_1 = action of acceptance of lot

a_2 = action of rejection of lot

$Ta_i(n,x,u)$ = total cost of taking action a_i, when after inspecting n items, x defectives have been found and u defectives are present in the uninspected portion of the lot

For the sample and lot disposition policy (ii), (iii) we have,

$$\begin{aligned} Ta_1 &= c_i\, n + c_r\, x + c_a\, u \\ Ta_2 &= c_i\, N + c_r\, x + c_r\, u \end{aligned} \qquad (2.38)$$

which can be written as,

$$Ta_i = K_i + C_i\, u$$

where K_i and C_i are constants not involving u.

Expected total cost expression is given by,

$$C_T\,(n) = E_x \min_a E_{u/x}\; Ta\,(n,\, x,\, u)$$

We seek to minimize the above cost expression.

$$C_T(n) = E_x \min_a E_{u/x} [K_i + C_i u]$$

$$= E_x \min_a [K_i + a\, E(u/x)] \qquad (2.39)$$

where,

$E(u/x)$ = Expected number of defectives remaining in the uninspected portion of lot given that x defectives are found in sample of size n.

From Eq. (2.39), it can be seen that optimal decision would be to accept the lot if,

$$K_1 + C_1 E(u/x) \leq K_2 + C_2 E(u/x) \qquad (2.40)$$

otherwise optimal decision is to reject the lot. It has been found in many cases $E(u/x)$ is a linear function of the number of defectives x, in the sample. In this case an expression $\phi(.)$ satisfying the relation, given below,

$$x = \phi(n, N, K_1, K_2, C_1, C_2, \boldsymbol{\rho}) \qquad (2.41)$$

can be found from Eq. (2.40) where $\boldsymbol{\rho}$ denotes vector containing parameters of conditional distribution of the number of defectives in the uninspected portion of the lot $h(u/x)$. In such a case the largest integer less than or equal to $\phi(.)$ will be the optimal choice for the acceptance number c. Once the acceptance number c has been obtained in terms of n, we can proceed to find the optimal sample size. Any standard integer search procedure can be used to find n which minimizes the

expected total cost equation given by,

$$C_T(n) = \sum_{x=0}^{c} (K_1 + C_1\, E(u/x))\, g_n(x)$$

$$+ \sum_{x=c+1}^{n} (K_2 + C_2\, E(u/x))\, g_n(x) \qquad (2.42)$$

where $g_n(x)$ is the unconditional distribution of the number of defectives x in the sample of size n.

Distributional Considerations:

Let,

$f_N(X)$ = Distribution of number of defectives X in the lot of size N.

$f_n(x/X)$ = Distribution of defectives x, in the sample of size n, given X defectives in the lot.

$$g_n(x) = \sum_{X=x}^{N-n+x} f_n(x/X) . f_N(X) \qquad (2.43)$$

If sampling is conducted without replacement $f_n(x/X)$ is hypergeometric, and is given by,

$$f_n(x/X) = f_n\,(x/x+u) = \frac{\binom{n}{x}\binom{N-n}{u}}{\binom{N}{x+u}} \qquad (2.44)$$

$$g_n(x) = \sum_{u=0}^{N-n} f_n(x/x+u)\ f_N(x+u) \qquad (2.45)$$

By Bayes theorem, the distribution of the number of defectives u (remaining in the uninspected portion) given x (the number of

defectives in the sample) can be expressed as,

$$h(u/x) = \frac{f_n(x/u+x) \cdot f_N(x+u)}{g_n(x)}$$

$$h(u/x) = \frac{\binom{n}{x}\binom{N-n}{u}}{\binom{N}{x+u}} \; \frac{f_N(u+x)}{g_n(x)} \qquad (2.46)$$

Worthram and Wilson (28) have shown that the expectation of u given x, E (u/x) can be expressed conveniently as,

$$E(u/x) = \sum_{u=0}^{N-n} u\, h(u/x) = (N-n)\, \frac{(x+1)}{(n+1)}\; \frac{g_{n+1}(x+1)}{g_n(x)} \qquad (2.47)$$

where $g_{n+1}(x+1)$ denotes unconditional distribution of x+1 defectives based on the sample of n+1 items.

Hald (19) made an observations which can be used to simplify calculation of E(u/x). He notes that if prior distribution $f_N(X)$, is of hypergeometric, Binomial, Polya, rectangular or mixed Binomial families, then the unconditional distribution of x, $g_n(x)$ will also be of the same family as that of prior. This general property is termed as reproducibility. For the case of reproducible distributions $g_n(x)$ assumes the same form as $f_N(X)$, the only difference being N and X are replaced by n and x.

Let $f_N(X)$ be Polya with parameters n and x ,

$$f_N(X) = \binom{N}{X} \frac{\lceil x' + X}{\lceil x'} \; \frac{\lceil n' + N - x' - X}{\lceil n' - x'} \; \frac{\lceil n'}{\lceil n' + N} \qquad (2.48)$$

Then by the property of reproducibility,

$$g_n(x) = \binom{n}{x} \frac{\lfloor x' + x}{\lfloor x'} \frac{\lfloor n' + n - x' - x}{\lfloor n' - x'} \frac{\lfloor n'}{\lfloor n' + n} \qquad (2.49)$$

Substituting for $g_n(x)$ and $g_{n+1}(x+1)$ in Eq. (2.47), we get,

$$E(u/x) = (N - n) \frac{(x' + x)}{(n' + n)} \qquad (2.50)$$

For a fixed value of n we have from Eq. (2.40),

$$c_i\, n + c_r\, x + c_a\, E(u/x) \leq c_i\, N + c_r\, x + c_r\, E(u/x) \qquad (2.51)$$

$$E(u/x) \leq \frac{c_i(N - n)}{(c_a - c_r)}$$

$$(N-n) \frac{(x + x')}{(n + n')} \leq \frac{c_i\,(N - n)}{(c_a - c_r)}$$

$$x \leq \frac{c_i\,(n + n')}{c_a - c_r} - x' \qquad (2.52)$$

The optimal acceptance number c can be given as the largest integer less than or equal to right hand side of Eq. (2.52),

$$c = \left[\frac{c_i\,(n + n')}{(c_a - c_r)} - x' \right] \qquad (2.53)$$

Model Formulation With Errors:

Let us now consider the situation when type I and type II errors are present in the inspection process. Expected number of the defectives observed in the sample of size n, will be given by,

$$\begin{aligned} x_e &= (n - x)\, e_1 + x(1 - e_2) \\ &= n\, e_1 + x(1 - e_1 - e_2) \\ &= n\, e_1 + x\, \varepsilon \end{aligned} \qquad (2.54)$$

where, $\varepsilon = 1 - e_1 - e_2$ = Probability of classifying an item correctly.

Let Ta_1' and Ta_2' be the costs of taking actions of accepting the lot and rejecting the lot respectively.

$$Ta_1' = c_i n + c_r x_e + c_a x e_2 + c_a u \qquad (2.55)$$

where,

$c_i\, n$ = Inspection cost

$c_r\, x_e$ = Repair cost of repairing x_e number of observed defectives

$x\, e_2$ = Expected number of defectives classified as good in sample.

$c_a\, x\, e_2$ = Cost of accepting defectives classified as good in the sample

$c_a\, u$ = Cost of accepting the defectives u present in the uninspected portion of lot,

$$Ta_2' = c_i N + c_r X_e + c_a x e_2 + c_a u e_2 \qquad (2.56)$$

Here the rejected portion of lot is subjected to 100 percent inspection so out of u defectives present, $u\, e_2$ are classified as good on an average and accepted. Hence the cost $c_a\, u\, e_2$. The other terms are same as before. The above equations can be written in terms of x and u as follows:

$$T'a_1 = (c_i + c_r e_1) n_1 + (c_r \varepsilon + c_a e_2) x + c_a u$$
$$T'a_2 = (c_i + c_r e_1) N + (c_r \varepsilon + c_a e_2) x + (c_r \varepsilon + c_a e_2) u \qquad (2.57)$$

Again, $T'a_i = K'_i + C'_i u$

where K'_i and C'_i are independent of u. The expected total cost is given by,

$$C'_T(n) = E_X \underset{a}{\mathrm{Min}}\ E_{u/x} [K'_i + C'_i u]$$
$$= E_X \underset{a}{\mathrm{Min}} [K'_i + C'_i E(u/x)]$$
$$= \sum_{x=0}^{c'} [K'_1 + C'_1 E(u/x)) g_n(x)$$
$$+ \sum_{x=c'+1}^{n} [K'_2 + C'_2 E(u/x)] g_n(x) \qquad (2.58)$$

The value of c' may be analogously obtained. c' is the largest integer less than or equal to $\phi(.)$ where $\phi(.)$ is defined as,

$$x \leq \phi(n, N, K'_1, K'_2, C'_1, C'_2, p) \qquad (2.59)$$

x_e and not x is the number of defectives the inspector observes. Thus when inspection errors are present, c is found by replacing x in equation (2.59), by x_e

$$x_e \leq \varepsilon\ \phi(.) + n e_1 \qquad (2.60)$$

Therefore acceptance number supplied to erroneous inspector is simply the largest integer less than or equal to the right hand side of Eq. (2.60).

In the presence of errors solving for $E(u/x)$, we will get,

$$x \leq \left[\frac{(c_i + c_r e_1)(n' + n)}{(c_a (1 - e_2) - c_r \varepsilon)} - x' \right] \tag{2.61}$$

Thus when inspection error is present c' is the largest integer less than or equal to right hand side of the eq.(2.61) Then acceptance number c is the largest integer less than or equal to the quantity $\phi\varepsilon + n e_1$ where ϕ is expression on right hand side of Eq. (2.61).

Numerical Example:

A numerical problem was solved. Following are the inputs taken from Collins et al (25). The results were computed for a couple of error pairs and hence the complete results shown are from Collins et al (25).

$c_i = 2.00$

$c_r = 1.90$

$c_a = 40.00$

$x = 3$

$n = 60$

$N = 1000$

Case 1: When errors are not present:

$Ta_1 = K_1 + C_1 u$

$Ta_2 = K_2 + C_2 u$

$K_1 = c_i n + c_r x,\qquad C_1 = c_a$

$K_2 = c_i N + c_r x,\qquad C_2 = c_r$

Substituting various values in Eq. (2.53), for a particular value of n we get c as,

$$c = \left[\frac{2.85 + n}{19.05} \right]$$

For this plan expected total cost was computed from Eq. (2.42). The optimal plan was obtained by using a search procedure with a step size, which can be adjusted. The following results were obtained.

Optimum sample size n = 122

Optimum acceptance number c = 6

Expected total cost per lot = 1745.61

This optimal plan is based on the assumption of perfect inspection. To have an idea of how this plan will behave in presence of errors few error pairs was considered. Let Δ_1 be defined as percentage error caused by using the optimal plan (found on basis of assumption of perfect inspection) when error is present.

$$\Delta_1 = \frac{C_{TE} - C_T}{C_T} \times 100 \qquad (2.62)$$

C_{TE} = Expected total cost of above plan when error are present.

The next step was to find an optimal sampling plan when errors are accounted for. The optimal plans were found for each error pair in the similar way as in the case error free inspection. Let the expected total cost of the optimal sampling plan when inspection errors are present be represented by C_T . Let Δ_2 be defined as,

$$\Delta_2 = \frac{C_T' - C_T}{C_T} \times 100 \qquad (2.63)$$

This is a measure of increase in the cost as percentage of the cost of optimal plan (with error free inspection). In a way this is the cost penalty of not using the information that inspection errors are present. There is another measure Δ_3 defined as,

$$\Delta_3 = \frac{C_{TE} - C_T'}{C_T'} \times 100 \qquad (2.65)$$

This measure gives an idea of increase in the cost if we do not use the optimal plan (found after accounting for errors) and instead use the plan (found with error free assumption).

The optimal sampling plans in the presence of error are given in Table 9. Δ_1, Δ_2 and Δ_3 are given for various error pairs in Tables 8, 9 and 10 respectively.

Optimal plan with error-free assumption, n = 122, c = 6

Optimal expected cost = 1745.61

Table 8: Expected cost of the optimal sampling plan (n = 122, 6) for selected error pairs.

Error pairs (e_1, e_2)	Expected total cost C_{TE}	Percentage error Δ_1
(0.00, 0.00)	1745.61	0.00
(0.00, 0.05)	1795.11	2.84
(0.00, 0.15)	1878.33	7.60
(0.03, 0.00)	1927.05	10.39
(0.03, 0.05)	1996.83	14.39
(0.03, 0.15)	2126.08	21.80
(0.10, 0.00)	2271.10	30.10
(0.10, 0.05)	2365.19	35.49
(0.10, 0.15)	2553.28	46.27

Table 9: Optimal sampling plans for selected error pairs.

Error-pair (e_1, e_2)	Optimal plan (n, c)	Expected total cost C_T	Percentage error Δ_2
(0.00, 0.00)	(122, 6)	1745.61	0.00
(0.00, 0.05)	(132, 7)	1793.92	2.77
(0.00, 0.15)	(180, 11)	1865.40	6.86
(0.03, 0.05)	(143, 12)	1794.68	2.81
(0.03, 0.15)	(200, 19)	1837.50	5.25
(0.03, 0.15)	(200, 19)	1898.29	8.74
(0.10, 0.00)	147, 24)	1883.14	7.88
(0.10, 0.05)	(160, 27)	1915.57	8.74
(0.10, 0.15)	(200, 35)	1953.20	11.89

Table 10: Percentage error realized by using plan (n = 122, c = 6) instead of optimal plan when inspection error is present.

Error pairs (e_1, e_2)	Expected Total Cost		Percentage error Δ_3
	C_{TE}	C_T	
(0.00, 0.00)	1745.61	1745.61	0.00
(0.00, 0.05)	1795.11	1793.92	0.07
(0.00, 0.15)	1878.33	1865.40	0.69
(0.03, 0.00)	1927.05	1794.68	7.37
(0.03, 0.05)	1996.83	1837.50	8.68
(0.03, 0.15)	2126.08	1898.29	11.99
(0.10, 0.00)	2271.10	1883.14	20.60
(0.10, 0.05)	2365.19	1915.57	23.47
(0.10, 0.15)	2553.28	1953.20	30.72

Tables 8, 9 and 10 have been taken from Collins et al (25). Δ_1, Δ_2 and Δ_3 are measures of cost effects of the errors. Δ_1 is the cost penalty of not having the information that inspection errors are present. Δ_2 is the cost penalty because of the presence of errors. Δ_3 is the cost penalty of not using the information that the inspection errors are present.

The results indicate that costs introduced through realistic errors in the inspection process can be appreciable. This indicates that some non trivial amount of money spent to improve inspection performance may be worthwhile.

In the above analysis it may be noted that, in the presence of errors, only random variable used in defectives present and not the defectives observed. To account for errors acceptance number c modified to equivalent expected acceptance number c'. This has worked out well in the present example and has not affected the results adversely, however the same pattern cannot be guaranteed in all cases. The exact analysis in the presence of errors is presented in the next chapter for the double sampling plan. The first sample of the double sample corresponds to the case of single sampling and hence in the case of single sampling the analysis will be similar to that in case of first sample of the double sampling plan.

2.6 Conclusions:

This chapter was devoted to an extensive study and analysis of effect of inspection errors on single sampling plan. A final over view is justified to recapitulate the important points.

1. The distribution of observed defectives also follows Binomial distribution (n, p_e) if actual defectives follow Binomial distribution (n, p).

2. In the presence of type II errors, apparent incoming quality is better where as in the presence of type I errors it is worse.

3. For a given single sampling plan the probability of acceptance increases in the presence of type II errors and decreases in the presence of type I errors.

4. The effect of type I errors on probability of acceptance is more profound because we usually deal with good quality i.e. $p < 0.20$ or so. This means on an average 80 percent of items are good. Type I error will lead to classification of some of these incorrectly.

5. The effect of e_1 is to improve outgoing quality whereas e_2 not only increases average outgoing quality, it also changes the shape of AOQ curve.

6. Average total inspection increases in presence of type I error and decreases in the presence of type II error.

7. Conventional concept of designing a plan based on AOQL and ATI is not useful in the presence of type II errors.

8. The AOQL and minimum ATI based design may however be carried out if either only e_1 or e_2 is present or the combination of errors is such that it does not lead to a continuously increasing AOQ curve.

9. The economic effects of e_1 and e_2 do more than, just showing the adverse effects of inspection errors. The effects justify a need to work for incorporating the inspection errors in quality control models.

CHAPTER III

INSPECTION ERRORS AND DOUBLE SAMPLING PLAN

3.1 Introduction:

A double sampling plan is designated by five numbers n_1, n_2, c_1, c_2 and c_3. c_1 being less than c_2 and c_2 being less than or equal to c_3. The plan works as follows: A sample of size n_1 is drawn from the lot, and inspected. If the number of defectives found is c_1 or less the lot is immediately accepted. If number of defectives found is c_2 or more the lot is immediately rejected. If number of defectives is more than c_1 but less than c_2, a second sample of size n_2 is drawn and inspected. If in the combined sample there are c_3 or less defectives, the lot is accepted otherwise it is rejected. Frequently c_2 is taken equal to c_3. In the following sections we will be considering plans of this simpler type ($c_3 = c_2$). Double sampling plan will thus be represented by four numbers n_1, n_2, c_1 and c_2.

In this chapter, we will consider the effect of inspection errors on various performance measures of a double sampling plan such as probability of acceptance, average sample number etc. Further we will attempt to devise a procedure to compensate for errors present, while designing a double sampling plan, we

will then analyze the AOQL and ATI based design in presence of errors. Finally a model will be presented to analyze the economic effects of inspection errors on double sampling plan.

3.2 Effect of Errors on Some Performance Measures:

3.2.1 Apparent Incoming Quality:

Similar to the case of single sampling plan, the erroneous inspector observes apparent incoming quality p_e instead of true incoming quality p. This is due to his tendency of making type I and type II errors. The apparent incoming quality is given by Eq. (2.1) as,

$$p_e = p(1 - e_2) + (1 - p)\, e_1$$

with usual notations (introduced in Chapter II).

3.2.2 Probability of Acceptance:

In case of a double sampling plan we have two probabilities of acceptance, one for acceptance on the first sample and another for acceptance on the second sample. The two are easily obtained by replacing p by p_e in equations for Pa without errors given in Duncan (5).

Pa_1 probability of acceptance on first sample is given by,

$$Pa_1 = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1} (1 - p)^{n_1-x_1} \qquad (3.1)$$

In presence of errors Eq. (3.1) can be written as,

$$Pa_{1e} = \sum_{x_1e=0}^{c_1} \binom{n_1}{x_{1e}} p_e^{x_{1e}} (1 - p_e)^{n_1 - x_{1e}} \quad (3.2)$$

Similarly probability of rejection in presence of and in absence of errors on first sample is given by, respectively,

$$Pr_1 = 1 - \sum_{x_1=0}^{c_2} \binom{n_1}{x_1} p^{x_1} (1 - p)^{n_1 - x_1} \quad (3.3)$$

$$Pr_{1e} = 1 - \sum_{x_{1e}=0}^{c_1} \binom{n_1}{x_{1e}} p_e^{x_{1e}} (1 - p_e)^{n_1 - x_{1e}} \quad (3.4)$$

Probability of acceptance on the second sample is given by,

$$Pa_2 = \sum_{x_1=c_1+1}^{c_2-1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1}$$

$$\cdot \sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x2} p^{x_2} (1-p)^{n_2-x_2} \quad (3.5)$$

$$Pa_{2e} = \sum_{x_{1e}=c_1+1}^{c_2-1} \binom{n_1}{x_{1e}} p_e^{x_1} (1-p_e)^{n_1-x_{1e}}$$

$$\cdot \sum_{x_{2e}=0}^{c_2-x_1e} \binom{n_2}{x_{2e}} p_e^{x_{2e}} (1-p_e)^{n_2-x_{2e}} \quad (3.6)$$

The probability of rejection on second sample is given by,

$$Pr_2 = 1 - Pa_1 - Pa_2 - Pr_1 \quad (3.7)$$

$$Pr_{2e} = 1 - Pae_1 - Pae_2 - Pre_1 \quad (3.8)$$

Finally total probability of acceptance and rejection is given by,

$$Pa_e = Pa_{1e} + Pa_{2e} \tag{3.9}$$

$$Pr_e = Pr_{1e} + Pr_{2e} \tag{3.10}$$

To study the effect of inspection errors on Pa a typical double sampling plan ($N = 1000$, $n_1 = 34$, $c_1 = 3$, $n_2 = 19$, $c_2 = 5$) found for ($AQL = 0.04$, $\alpha = 0.05$), ($RQL = 0.20$, $\beta = 0.10$) requirements was considered. For this plan and few representative error pairs probability of acceptance was computed at selected incoming quality levels. The results are illustrated in Tables 11 and 12. Table 11 gives the probability of acceptance (total) in presence of errors. Table 12 shows the percentage change of probability of acceptance from that with error-free inspection i.e. $\frac{Pa_e - Pa}{Pa} \times 100$. Fig. 6 shows the effect of errors on the OC curve. Another plan ($N = 1000$, $n_1 = 70$, $c_1 = 1$, $n_2 = 59$, $c_2 = 3$) found for ($AQL = 0.01$, $\alpha = 0.05$), ($RQL = 0.07$, $\beta = 0.05$) requirements is considered. A few representative error pairs are considered.

Tables 11 and 12 and the Fig. 6 show the similar effects as in case of single sampling plan. Probability of acceptance increases in presence of type II errors due to erroneous classification of defective items as good. The effect is more at higher value of p because the number of defectives present which can be erroneously classified is higher. The effect of type I errors is also similar to the case of single sampling plan. Pa_e is less than Pa because of incorrect classification of good items.

TABLE 11: PROBABILITY OF ACCEPTENCE IN PRESENCE OF ERRORS(DOUBLE SAMPLING PLAN)

PLAN: (N=1000,n1=34,c1=3,n2=19,c2=5)

(AQL=0.04,α=0.05,RQL=0.20,β=0.10)

	ERROR-PAIRS (e1,e2)	INCOMING QUALITY (p) 0.010	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	0.99967	0.99104	0.92101	0.77492	0.59094	0.41274	0.28663	0.16048	0.09045
2	(0.000,0.050)	0.99973	0.99248	0.93178	0.80044	0.62849	0.45496	0.30633	0.19317	0.11463
3	(0.000,0.075)	0.99975	0.99314	0.93685	0.81283	0.64727	0.47677	0.32754	0.21128	0.12857
4	(0.000,0.100)	0.99978	0.99375	0.94171	0.82493	0.66602	0.49899	0.34965	0.23061	0.14383
5	(0.000,0.125)	0.99980	0.99433	0.94637	0.83674	0.68466	0.52157	0.37263	0.25177	0.16048
6	(0.000,0.150)	0.99982	0.99487	0.95081	0.84623	0.70318	0.54447	0.39643	0.27299	0.17859
7	(0.000,0.200)	0.99986	0.99585	0.95904	0.87017	0.73961	0.59094	0.44636	0.31037	0.21941
8	(0.010,0.000)	0.99593	0.97356	0.87299	0.70670	0.52330	0.35721	0.22607	0.13375	0.07429
9	(0.025,0.000)	0.97356	0.92377	0.78353	0.60500	0.42939	0.28273	0.17392	0.10045	0.05466
10	(0.050,0.000)	0.87299	0.78353	0.60970	0.43784	0.29268	0.18336	0.10914	0.06024	0.03176
11	(0.075,0.000)	0.70870	0.60500	0.43784	0.29606	0.18821	0.11296	0.06423	0.03467	0.01777
12	(0.100,0.000)	0.52431	0.42939	0.29268	0.18621	0.11463	0.06632	0.03651	0.01914	0.00957
13	(0.010,0.100)	0.99680	0.97917	0.89962	0.76272	0.59844	0.43786	0.30083	0.19518	0.12005
14	(0.025,0.100)	0.97591	0.93432	0.81699	0.66133	0.49859	0.35342	0.23635	0.14991	0.09045
15	(0.100,0.100)	0.53161	0.44636	0.32037	0.21941	0.19363	0.09045	0.05466	0.03176	0.01777

TABLE 12: PERCENTAGE CHANGE IN PROBABILITY OF ACCEPTANCE FOR VARIOUS ERROR-PAIRS (DOUBLE SAMPLING PLAN)

PLAN: (N=1000, n1=34, c1=3, n2=19, c2=5)

(AQL=0.04, α=0.05, RQL=0.20, β=0.10)

	ERROR-PAIRS	INCOMING QUALITY 'p'								
	(e1,e2)	0.01	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	0.006	0.145	1.17	3.29	6.35	10.23	14.89	20.37	26.73
3	(0.000,0.075)	0.008	0.212	1.72	4.89	9.53	15.51	22.63	31.66	42.14
4	(0.000,0.100)	0.011	0.273	2.25	6.45	12.71	20.90	31.14	43.70	59.02
5	(0.000,0.125)	0.013	0.332	2.75	7.98	15.66	26.37	39.76	56.51	77.42
6	(0.000,0.150)	0.015	0.386	3.24	9.46	18.94	31.52	46.68	70.11	97.45
7	(0.000,0.200)	0.019	0.485	4.13	12.29	25.16	43.17	67.41	93.40	142.58
8	(0.010,0.000)	-0.374	-1.76	-5.21	-8.55	-11.45	-13.35	-15.21	-16.66	-17.87
9	(0.025,0.000)	-2.61	-6.79	-14.93	-21.93	-27.34	-31.50	-34.77	-37.41	-39.57
10	(0.050,0.000)	-12.67	-20.94	-33.80	-43.50	-50.47	-55.57	-59.44	-62.46	-64.89
11	(0.075,0.000)	-29.10	-38.95	-52.46	-61.79	-68.15	-72.63	-75.91	-78.40	-80.35
12	(0.100,0.000)	-47.55	-58.67	-68.22	-75.71	-80.60	-83.93	-86.31	-88.07	-89.42
13	(0.010,0.100)	-0.31	-1.20	-2.32	-1.57	1.27	6.09	12.83	21.62	32.73
14	(0.025,0.100)	-2.38	-5.72	-11.29	-14.56	-15.56	-14.37	-11.36	-6.59	0.00
15	(0.100,0.100)	-46.82	-54.95	-65.22	-71.69	-75.66	-78.09	-79.50	-80.21	-80.35

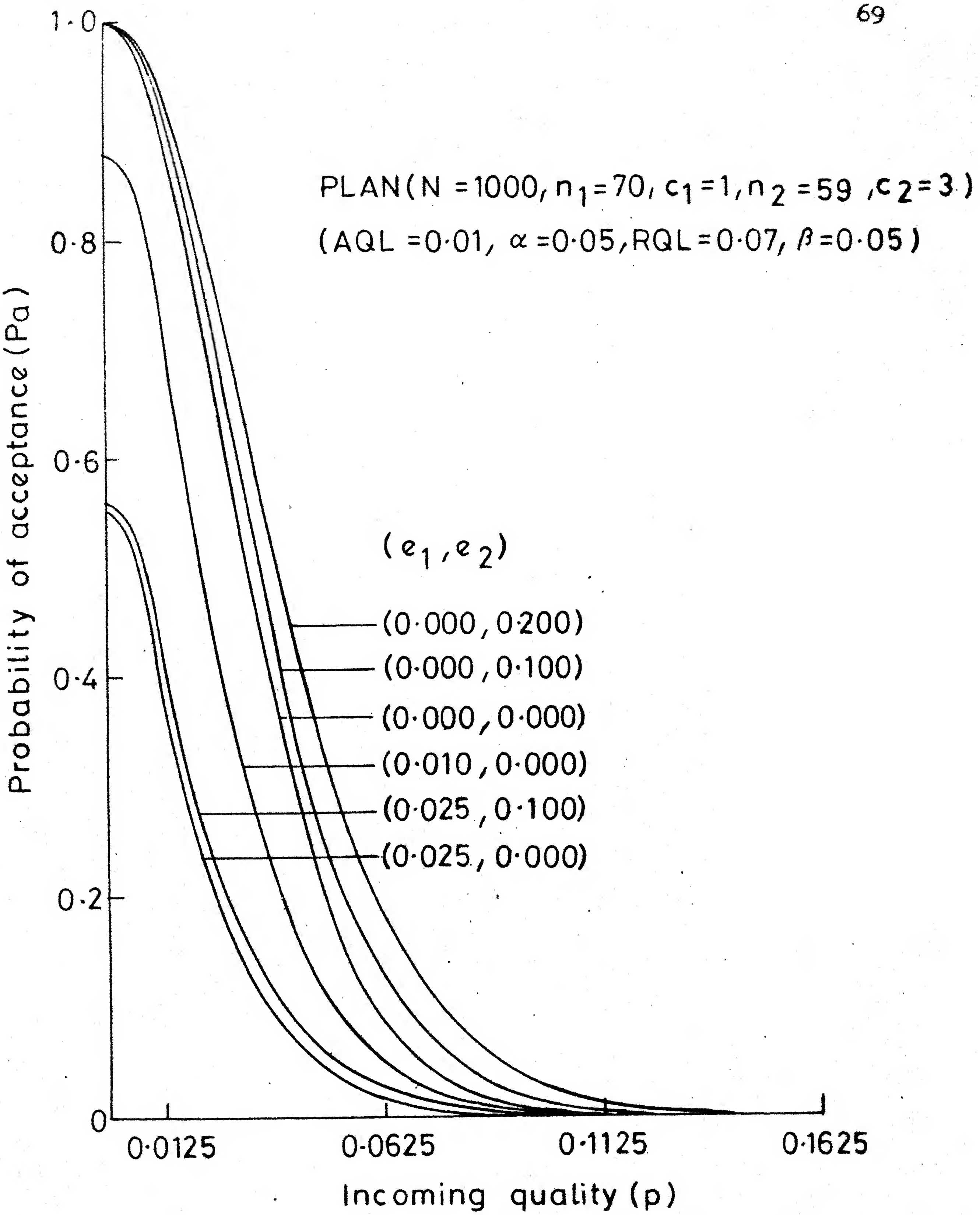

Fig. 6 OC Curve in presence of inspection errors (Double sampling plan)

3.2.3 Average Outgoing Quality (AOQ).

Average outgoing quality is a useful measure with rectifying inspection. We have nine possible rectification policies given by Worthram and Mogg (27). These policies include three different lot dispositions and three sample disposition as discussed in Chapter II. We will consider two of the rectification policies:

1. Defectives found both in the sample and in the rejected lots which are subjected to 100 percent inspection are not replaced.

2. Defectives found both in the sample and in the rejected lots which are put to 100 percent inspection are replaced by good items.

Average outgoing quality for the above two rectification policies in absence of error is given by [Duncan (5)].

Case I: Rectification Without Replacement:

In case of policy of rectification when defectives found are not replaced but scrapped, AOQ is given by,

$$AOQ = \frac{[Pa_1\ (N - n_1) + Pa_2\ (N - n_1 - n_2)]\ p}{N - n_1p - Pr_1\ (N - n_1)p - n_2p(1 - Pa_1 - Pr_1) - p(N - n_1 - n_2)\,Pr_2} \tag{3.11}$$

where, n_1p = Expected number of defectives removed in the first sample

$n_2p(1-Pa_1-Pr_1)$ = Expected number of defectives removed in second sample if one is drawn.

$p(N-n_1)\ Pr_1$ = Expected number of defectives removed if lot is rejected on the first sample and is inspected 100 percent.

The other terms are self explanatory.

Case II: Rectification with Replacement:

AOQ is given by,

$$AOQ = \frac{[(N - n_1)\ Pa_1 + (N - n_1 - n_2)\ Pa_2]\ p}{N} \qquad (3.12)$$

In expressions (3.11) and (3.12) have to be modified to take into account the presence of errors. Let us consider the ways, by which defectives can appear in the finally accepted lot.

The expected number of defectives remaining in the uninspected portion of a lot, accepted on first or second sample is given by A_1 and A_2 respectively,

$$\begin{aligned} A_1 &= p\ (N - n_1)\ Pa_{1e} \\ A_2 &= p\ (N - n_1 - n_2)\ Pa_{2e} \end{aligned} \qquad (3.13)$$

The expected number of defectives classified as good in first and second sample is given by B_1 and B_2 respectively,

$$\begin{aligned} B_1 &= n_1\ p\ e_2 \\ B_2 &= n_2\ p\ e_2\ (1 - Pa_{1e} - Pr_{1e}) \end{aligned} \qquad (3.14)$$

The expected number of defectives introduced through replacement items required to replenish the first and second sample is given by C_1 and C_2 respectively,

$$C_1 = \frac{n_1 p_e}{1-p_e} pe_2$$

$$C_2 = \frac{n_2 p_e}{1-p_e} pe_2 (1 - Pa_{1e} - Pr_{1e}) \tag{3.15}$$

Thus expected number of defectives introduced through first and second sample through inspection of samples and replacement parts to replenish the samples is given by D_1 and D_2 respectively ($D_i = B_i + C_i$).

$$D_1 = \frac{n_1 pe_2}{1-p_e}$$

$$D_2 = \frac{n_2 pe_2}{1-p_e} (1 - Pa_{1e} - Pr_{1e}) \tag{3.16}$$

Similarly the expected number of defectives introduced through inspection of rejected lot on first or second sample, and the replacement items to replenish the lot is given by E_1 and E_2 respectively,

$$E_1 = \frac{(N - n_1)\ p\ e_2}{1 - p_e} Pr_{1e}$$

$$E_2 = \frac{(N - n_1 - n_2)\ pe_2}{1 - p_e} Pr_{2e} \tag{3.17}$$

Note that in the above analysis, it has been assumed that the replacement items are also inspected. Let us now consider the two rectification policies.

Case 1: Defectives found in sample and screened lot not replaced.

Total expected number of defectives Zd_1 in the finally accepted lot from the above equations. (denominator $(1-p_e)$ will not be present because there is no replacement).

$$Zd_1 = n_1pe_2 + n_2pe_2 (1-Pa_{1e} - Pr_{1e}) + p(N-n_1)Pa_{1e}$$
$$+ p(N-n_1-n_2) Pa_{2e} + p(N-n_1)e_2 Pr_{2e}$$
$$+ p(N-n_1-n_2) e_2 Pr_{2e} \tag{3.18}$$

Expected total number of items Z_n in the finally accepted lot is given by,

$$Z_n = N-n_1 p_e - n_2p_e (1-Pa_{1e} - Pr_{1e}) - p_e(N-n_1) Pr_{1e}$$
$$- p_e (N-n_1-n_2) Pr_{2e} \tag{3.19}$$

AOQ_e is then given by,

$$AOQ_e = \frac{Zd_1}{Z_n} \tag{3.20}$$

Case 2: Defectives found in the sample and in the screened portion of lot are replaced.

Total expected number of defectives Zd_2 in the finally accepted lot from Eq. (3.13), (3.16), and (3.17) is given by,

$$Zd_2 = \frac{n_1 pe_2}{1 - p_e} + \frac{n_2 pe_2}{1 - p_e} (1 - Pa_{1e} - Pr_{1e})$$
$$+ p(N - n_1) Pa_{1e} + p(N - n_1 - n_2) Pa_{2e}$$
$$+ p \frac{(N-n_1)e_2}{1 - p_e} Pr_{1e} + \frac{p(N-n_1-n_2) e_2}{1 - p_e} Pr_{2e} \tag{3.21}$$

AOQ_e is then given by,

$$AOQ_e = \frac{Zd_2}{N} \tag{3.22}$$

To illustrate the effect of inspection errors on AOQ, Case 2 i.e. rectification policy with replacement is considered. A typical double sampling plan ($N = 1000$, $n_1 = 34$, $c_1 = 3$, $n_2 = 19$, $c_2 = 5$) is considered. Table 13 gives value of AOQ for selected incoming quality levels and several error pairs. Table 14 shows the percentage change in AOQ i.e. $[(AOQ_e - AOQ)/AOQ]\ 100$. The values in Tables 13 and 14 show a pattern similar to that in the case of single sampling plan. Fig. 7 shows the AOQ curve versus incoming quality. The curves are plotted for plan ($N = 1000$, $n = 70$, $c_1 = 1$, $n_2 = 59$, $c_2 = 3$) and for few representative error pairs.

Type II error increases the AOQ values because defectives are being erroneously classified as good and are being accepted. The shape of the curve is same as in the case of single sampling plan. Type I error has an effect of reducing AOQ values for all incoming quality levels. The combined effect in presence of both errors, is more significant. The AOQ may be continuously rising for few typical error pairs.

2.2.4 Average Sample Number:

In case of a single sampling plan the sample size is fixed. But in double sampling plan the total size of sample to be inspected is not fixed. It depends upon whether the

TABLE 13:AVERAGE OUTGOING QUALITY IN PRESENCE OF VARIOUS ERRORS(DOUBLE SAMPLING PLAN)

PLAN: (N=1000,n1=34,c1=3,n2=19,c2=5)

(AQL=0.04,α=0.05,RQL=0.20,β=0.10)

	ERROR-PAIRS	INCOMING QUALITY (p)								
	(e1,e2)	0.010	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	9.656	23.933	44.476	56.199	57.015	49.739	38.525	27.030	17.396
2	(0.000,0.050)	9.675	24.021	15.260	58.876	62.820	55.318	50.438	41.088	33.040
3	(0.000,0.075)	9.684	24.062	45.617	60.161	65.554	63.199	56.285	48.076	40.869
4	(0.000,0.100)	9.692	24.101	45.952	61.375	68.200	67.463	62.044	55.023	48.696
5	(0.000,0.125)	9.701	24.139	46.265	62.523	70.726	71.693	67.705	61.918	56.514
6	(0.000,0.150)	9.710	24.175	46.558	63.694	73.137	75.512	73.256	68.749	64.314
7	(0.000,0.200)	9.727	24.244	47.084	65.572	77.613	83.209	83.977	82.158	79.814
8	(0.010,0.000)	9.621	23.510	42.152	51.303	50.572	43.035	32.657	22.523	14.285
9	(0.025,0.000)	9.404	22.305	37.823	43.781	41.409	34.046	25.112	16.908	10.505
10	(0.050,0.000)	8.430	18.911	29.414	31.662	28.199	22.064	15.604	10.134	6.102
11	(0.075,0.000)	6.840	14.594	21.108	21.796	18.119	13.585	9.263	5.627	3.412
12	(0.100,0.000)	5.357	10.350	14.099	13.589	11.028	7.969	5.261	3.217	1.837
13	(0.010,0.100)	9.655	23.785	14.131	57.365	62.430	60.989	55.913	49.919	44.886
14	(0.025,0.100)	9.486	22.817	40.571	50.850	53.970	52.099	47.875	43.467	40.222
15	(0.100,0.100)	5.671	12.375	19.455	22.898	24.346	25.139	26.100	27.663	29.978

NOTE: VALUES GIVEN IN TABLE ARE AOQ*1000

TABLE 14: PERCENTAGE CHANGE IN AOQ FOR VARIOUS ERROR-PAIRS(DOUBLE SAMPLING PLAN)

PLAN: (N=1000,n1=34,c1=3,n2=19,c2=5)

(AQL=0.04,α=0.05,RQL=0.20,β=0.10)

	ERROR-PAIRS	INCOMING QUALITY "p"								
	(e1,e2)	0.010	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	0.18	0.37	1.76	4.94	10.16	13.25	30.92	52.01	89.93
3	(0.000,0.075)	0.28	0.54	2.57	7.22	15.03	27.05	46.10	77.86	134.93
4	(0.000,0.100)	0.37	0.70	3.32	9.32	19.52	35.63	61.05	103.56	179.92
5	(0.000,0.125)	0.46	0.86	4.02	11.43	24.05	43.96	75.74	129.07	224.86
6	(0.000,0.150)	0.55	1.01	4.68	13.35	28.26	52.02	90.15	154.34	269.70
7	(0.000,0.200)	0.73	1.30	5.86	16.86	36.13	67.29	117.98	203.95	358.80
8	(0.010,0.000)	-0.37	-1.77	-5.23	-6.57	-11.30	-13.48	-15.23	-16.68	-17.88
9	(0.025,0.000)	-2.62	-5.30	-14.96	-21.97	-27.39	-31.55	-34.82	-37.45	-39.61
10	(0.050,0.000)	-12.70	-20.48	-33.87	-43.57	-50.54	-55.54	-59.50	-62.51	-64.92
11	(0.075,0.000)	-29.17	-39.02	-52.54	-61.31	-68.22	-72.69	-75.96	-78.44	-80.39
12	(0.100,0.000)	-47.63	-56.75	-63.30	-75.78	-80.65	-83.98	-86.34	-88.10	-89.44
13	(0.010,0.100)	0.09	-5.52	-9.77	2.24	9.57	22.52	45.13	84.68	158.02
14	(0.025,0.100)	-1.77	-4.57	-8.78	-9.37	-5.34	4.75	24.27	60.81	131.21
15	(0.100,0.100)	-11.24	-48.28	-56.26	-59.21	-57.30	-45.16	-32.25	2.34	72.32

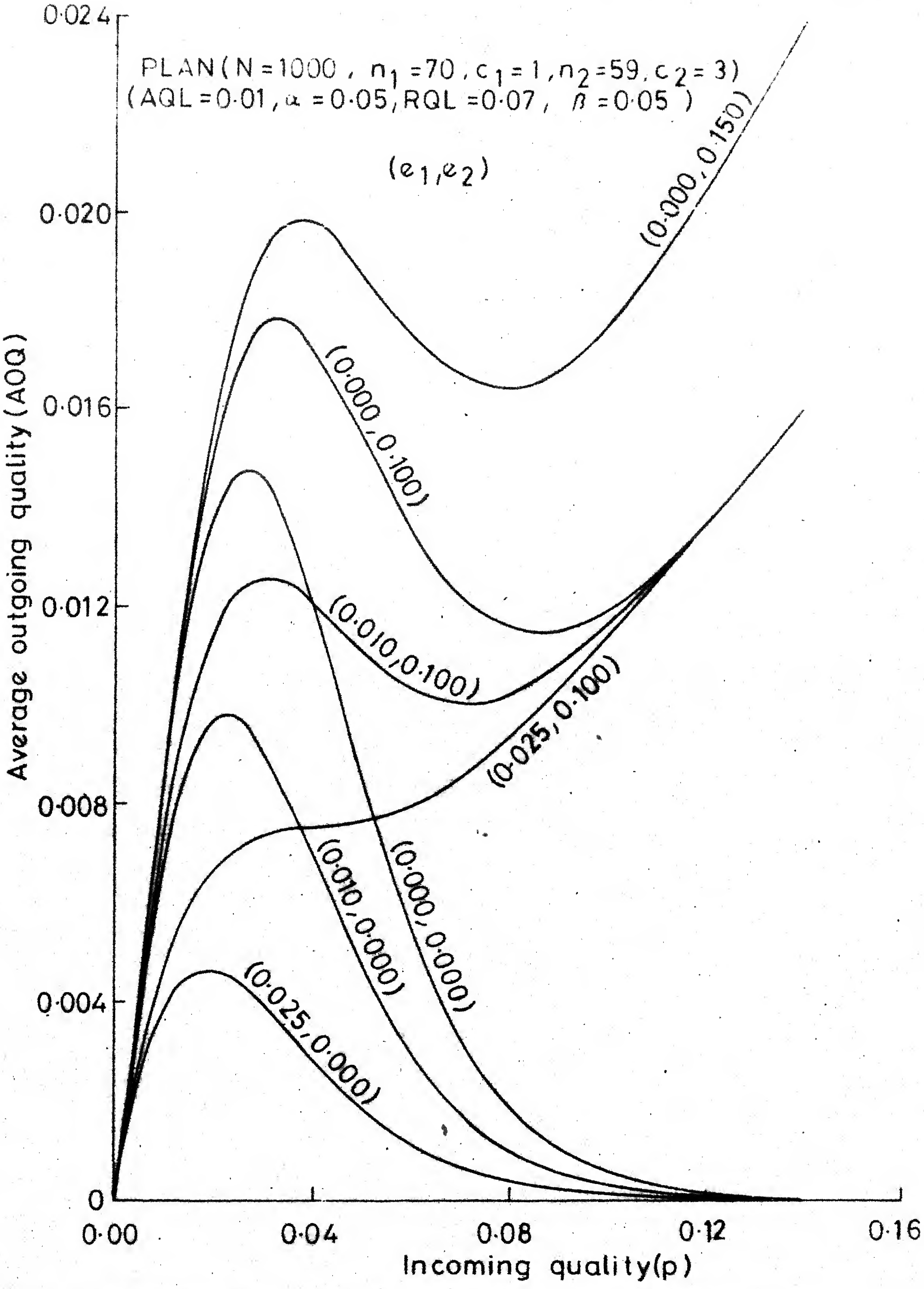

Fig. 7 Average outgoing quality in presence of inspection errors (Double sampling plan)

second sample is taken, which in turn depends upon incoming quality. Besides OC curve a double sampling has an average sample number (ASN) curve. ASN in absence of inspection errors is given by [Duncan (5)].

$$ASN = n_1 + n_2 (1 - Pa_1 - Pr_1) \tag{3.23}$$

In presence of errors Eq. (3.23) can be modified as,

$$ASN_e = n_1 + n_2 (1 - Pa_{1e} - Pr_{1e}) \tag{3.24}$$

To illustrate the effect of inspection errors same plan ($N = 1000$, $n_1 = 34$, $c_1 = 3$, $n_2 = 19$, $c_2 = 5$) is considered. For this plan and selected values of incoming quality, various error pairs were considered and ASN_e was computed. The results are shown in Tables 15 and 16. Table 15 shows the ASN_e values and Table 16 shows the percentage change from that with perfect inspection i.e. $[(ASN_e-ASN)/ASN]\ 100$. The point to be noted is ASN_e is computed assuming no curtailment of inspection. ASN depends upon Pa_{1e} as well as Pr_{1e} (Eq. 3.24). In presence of Type II errors, Pa_{1e} increases and Pr_{1e} decreases. Reverse is the effect in case of type I error. The two effects together determine the change in ASN. Hence the percentage change shown in Table 16 for type II error, is both negative and positive at different incoming quality levels. The Fig. 8 also shows the variation of ASN with incoming quality for a typical plan ($N = 1000$, $n_1 = 70$, $c_1 = 1$, $n_2 = 59$, $c_2 = 3$) for few representative error pairs.

TABLE 15: AVERAGE SAMPLING NUMBER IN PRESENCE OF ERRORS(DOUBLE SAMPLING PLAN)

PLAN: (N=1000,n1=34,c1=3,n2=19,c2=5)

(AQL=0.04,α=0.05,RQL=0.20,β=0.10)

	ERROR-PAIRS	INCOMING QUALITY (p)								
	(e1,e2)	0.01	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	34.01	34.19	35.56	38.00	40.23	41.27	41.01	39.65	38.36
2	(0.000,0.050)	34.01	34.16	35.35	37.61	39.85	41.14	41.21	40.32	38.97
3	(0.000,0.075)	34.01	34.15	35.26	37.42	39.65	41.04	41.27	40.54	39.27
4	(0.000,0.100)	34.00	34.13	35.16	37.22	39.44	40.92	41.31	40.73	39.57
5	(0.000,0.125)	34.00	34.12	35.08	37.03	39.22	40.78	41.32	40.90	39.85
6	(0.000,0.150)	34.00	34.11	34.99	36.84	38.99	40.62	41.30	41.05	40.13
7	(0.000,0.200)	34.00	34.09	34.83	36.47	38.50	40.23	41.18	41.25	40.62
8	(0.010,0.000)	34.03	34.54	36.42	38.92	40.75	41.32	40.69	39.37	37.87
9	(0.025,0.000)	34.54	35.51	37.87	40.09	41.23	41.10	40.05	38.63	37.22
10	(0.050,0.000)	35.42	37.37	40.05	41.21	41.15	40.19	38.82	37.42	36.25
11	(0.075,0.000)	38.42	40.09	41.21	41.17	40.26	38.93	37.56	36.39	35.49
12	(0.100,0.000)	40.75	41.23	41.15	40.26	38.97	37.63	36.47	35.58	34.95
13	(0.010,0.100)	34.07	34.43	35.95	38.18	40.16	41.21	41.19	40.35	39.09
14	(0.025,0.100)	34.49	35.30	37.35	39.49	40.92	41.31	40.78	39.67	38.36
15	(0.100,0.100)	40.71	41.18	41.25	40.62	39.57	38.36	37.22	36.25	35.49

TABLE 16: PERCENTAGE CHANGE IN ASN FOR VARIOUS ERROR-PAIRS(DOUBLE SAMPLING PLAN)

PLAN: (N=1000, n1=34, c1=3, n2=19, c2=5)

(AQL=0.04, α=0.05, RQL=0.20, β=0.10)

	ERROR-PAIRS	INCOMING QUALITY 'p'								
	(e1,e2)	0.010	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	0.00	-0.09	-0.59	-1.03	-0.92	-0.31	0.49	1.18	1.59
3	(0.000,0.075)	0.00	-0.15	-0.84	-1.53	-1.44	-0.55	0.63	1.73	2.37
4	(0.000,0.100)	-0.03	-0.18	-1.12	-2.05	-1.96	-0.85	0.73	2.21	3.15
5	(0.000,0.125)	-0.03	-0.20	-1.35	-2.55	-2.46	-1.19	0.75	2.63	3.88
6	(0.000,0.150)	-0.03	-0.23	-1.60	-3.05	-3.08	-1.57	0.71	3.01	4.61
7	(0.000,0.200)	-0.03	-0.29	-2.05	-4.03	-4.30	-2.52	0.41	3.51	5.89
8	(0.010,0.000)	0.21	1.02	2.42	2.42	1.32	0.12	-0.78	-1.20	-1.23
9	(0.025,0.000)	1.56	3.83	6.50	5.50	2.49	0.41	-2.30	-3.06	-2.97
10	(0.050,0.000)	7.09	10.76	12.63	8.45	2.29	-2.62	-5.34	-6.10	-5.50
11	(0.075,0.000)	14.44	17.25	15.89	8.34	0.07	-5.67	-8.42	-8.68	-7.48
12	(0.100,0.000)	19.85	20.59	15.72	5.95	-3.13	-8.82	-11.07	-10.72	-8.89
13	(0.010,0.100)	0.18	0.70	1.10	0.47	-0.17	-0.14	0.44	1.25	1.90
14	(0.025,0.100)	1.41	3.25	5.63	3.92	1.72	0.10	-0.56	-0.95	0.00
15	(0.100,0.100)	19.70	20.44	16.00	6.89	-1.64	-7.05	-9.24	-9.03	-7.48

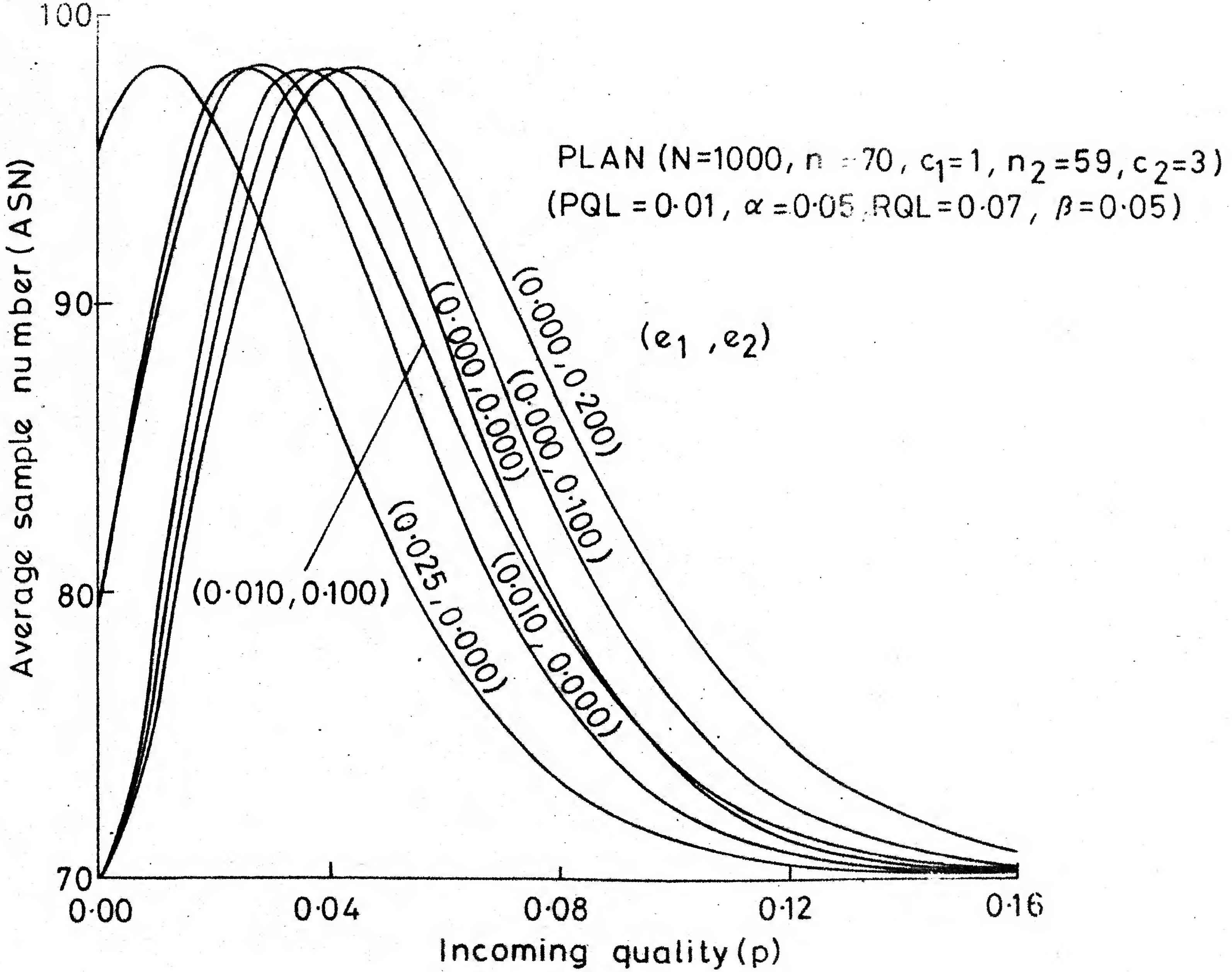

Fig. 8 Average sample number in presence of inspection errors (Double sampling plan)

2.2.5 Average Total Inspection (ATI):

Average total inspection in the presence of inspection errors for the two rectification policies considered in Sec. 2.2.3 is given by,

$$ATI_e = n_1 + n_2 (1 - Pa_{1e} - Pr_{1e}) + (N - n_1) Pr_{1e}$$
$$+ (N - n_1 - n_2) Pr_{2e} \qquad (3.25)$$

Eq. (3.25) is for the case of rectification without replacement. All the terms on right hand side are self explanatory. In case of rectification with replacement ATI_e is given by,

$$ATI_e = \frac{n_1+n_2 (1-Pa_{1e} - Pr_{1e}) + (N-n_1) Pr_{1e} + (N-n_1-n_2) Pr_{2e}}{(1 - p_e)} \qquad (3.26)$$

To illustrate the effect of inspection errors on ATI, same plan is considered ($N = 1000$, $n_1 = 34$, $c_1 = 3$, $n_2 = 19$, $c_2 = 5$). The rectification policy with replacement is assumed. The Tables 17 and 18 illustrate the effect of inspection errors on ATI. Table 17 shows ATI_e values at selected incoming quality levels for various error pairs. The Table 18 shows the percentage change i.e. $[(ATI_e-ATI)/ATI]$ 100. The results are similar to the case of single sampling plan. ATI increases in presence of type I errors and decreases in presence of type II errors. Rectification with replacement and replacement itemsbeing inspected may frequently lead to ATI values greater than the lot size.

TABLE 17: AVERAGE TOTAL INSPECTION IN PRESENCE OF ERRORS(DOUBLE SAMPLING PLAN)

PLAN: (N=1000,n1=34,c1=3,n2=19,c2=5)

(AQL=0.04,α=0.05,RQL=0.20,β=0.10)

	ERROR-PAIRS	INCOMING QUALITY 'p'								
	(e1,e2)	0.01	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	34.67	43.77	116.29	272.30	477.61	661.10	874.31	1024.90	1141.27
2	(0.000,0.050)	34.59	42.29	105.04	244.62	434.86	636.94	821.98	976.35	1098.42
3	(0.000,0.075)	34.56	41.61	99.76	231.24	413.64	610.85	794.02	950.41	1075.21
4	(0.000,0.100)	34.53	40.97	94.69	218.21	392.59	584.50	766.50	923.34	1050.72
5	(0.000,0.125)	34.50	40.37	89.66	205.54	371.74	557.94	737.65	895.14	1024.90
6	(0.000,0.150)	34.47	39.81	85.25	193.25	351.15	531.23	708.13	865.81	997.69
7	(0.000,0.200)	34.41	38.79	76.73	169.88	310.92	477.61	647.28	803.82	838.95
8	(0.010,0.000)	38.71	61.75	156.89	345.03	554.74	756.96	929.63	1066.79	1172.44
9	(0.025,0.000)	61.74	113.40	262.94	461.54	667.81	852.90	1094.63	1123.09	1214.71
10	(0.050,0.000)	106.89	202.94	456.20	657.56	839.78	950.67	1109.57	1202.03	1275.64
11	(0.075,0.000)	345.03	461.51	657.58	835.36	938.55	1101.23	1193.32	1266.77	1328.30
12	(0.100,0.000)	554.74	667.81	839.78	983.55	1098.42	1168.86	1261.34	1322.04	1376.13
13	(0.010,0.100)	38.01	55.97	136.57	285.61	469.04	657.56	829.14	973.44	1089.28
14	(0.025,0.100)	59.33	102.37	226.86	397.83	584.59	761.74	915.40	1041.20	1141.27
15	(0.100,0.100)	546.20	647.28	893.82	938.95	1050.72	1141.27	1214.71	1275.64	1328.30

TABLE 18: PERCENTAGE CHANGE IN ATI FOR VARIOUS ERROR-PAIRS (DOUBLE SAMPLING PLAN)

PLAN: (N=1000,n1=34,c1=3,n2=19,c2=5)

(AQL=0.04,α=0.05,RQL=0.20,β=0.10)

		INCOMING QUALITY (P)								
	ERROR-PAIRS	0.010	0.025	0.050	0.075	0.100	0.125	0.150	0.175	0.200
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	-0.23	-3.36	-9.67	-10.17	-8.95	-6.48	-5.99	-1.74	-3.75
3	(0.000,0.075)	-0.32	-4.93	-14.21	-15.08	-13.39	-10.31	-9.11	-7.27	-5.79
4	(0.000,0.100)	-0.40	-6.40	-18.57	-19.86	-17.80	-14.18	-12.33	-9.91	-7.93
5	(0.000,0.125)	-0.49	-7.77	-22.73	-24.52	-22.17	-18.16	-15.63	-12.66	-10.20
6	(0.000,0.150)	-0.58	-9.05	-26.69	-29.03	-26.48	-22.00	-19.01	-15.52	-12.58
7	(0.000,0.200)	-0.75	-11.36	-34.02	-37.61	-34.90	-29.86	-25.97	-21.57	-17.73
8	(0.010,0.000)	11.65	41.06	43.51	26.71	16.15	11.14	6.33	4.09	2.73
9	(0.025,0.000)	78.08	159.08	126.11	69.50	39.62	25.22	14.91	9.58	6.43
10	(0.050,0.000)	381.37	500.73	292.30	141.49	75.63	45.15	26.91	17.28	11.77
11	(0.075,0.000)	895.18	954.17	465.47	206.78	105.93	61.68	36.49	23.60	16.39
12	(0.100,0.000)	1500.06	1425.73	622.14	261.20	129.98	74.55	44.27	28.99	20.58
13	(0.010,0.100)	9.53	27.87	19.16	4.89	-1.70	-3.45	-5.17	-5.02	-4.56
14	(0.025,0.100)	71.13	133.83	95.08	46.10	22.38	11.84	4.70	1.59	0.00
15	(0.100,0.100)	1475.43	1378.82	591.22	244.82	120.06	67.55	38.93	24.46	16.39

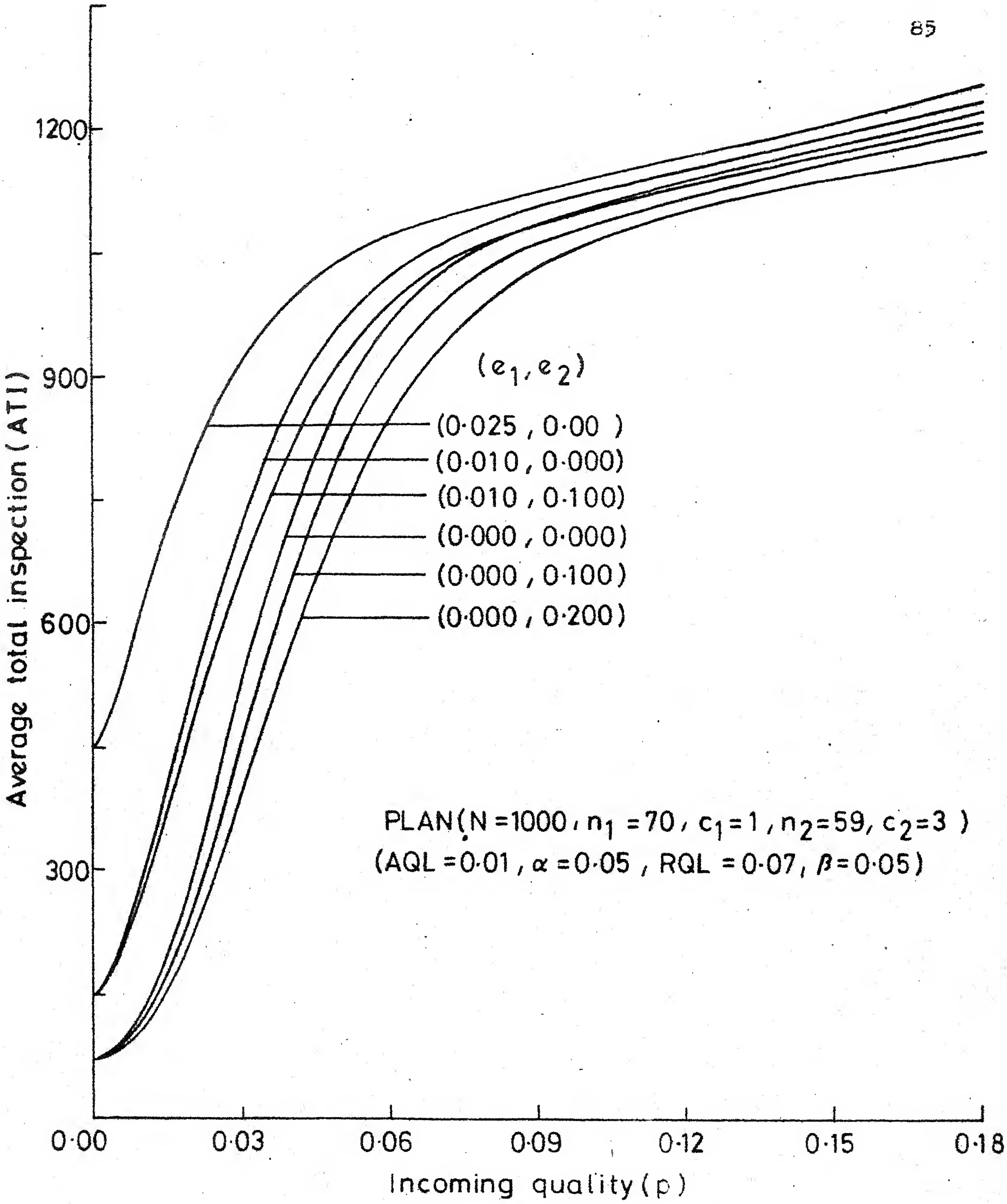

Fig. 9 Average total inspection in presence of inspection errors (Double sampling plan)

ATI value also depend on probability of acceptance Pa_{1e}, Pa_{2e} and probability of rejection Pr_{1e}, Pr_{2e} . The percentage change is therefore maximum at some intermediate incoming quality level. The Fig. 9 shows the variation of average total inspection in presence of errors with incoming quality. Few representative error pairs are considered.

3.3 Design of Double Sampling Plan Based Upon (AQL, α) and (RQL, β) Requirements:

It may be recalled that a double sampling plan is characterized by four numbers n_1, c_1, n_2 and c_2. The requirements (AQL, α), (RQL, β) lead to two equations in these four decision variables. (The exact solution, is not possible even in case of two decision variables as seen for a single sampling plan). Therefore, design for a double sampling plan is an open one with lot of subjectivity involved. We can have a large set of double sampling plans, all of which will satisfy (AQL, α), (RQL, β) requirements. But to analyze the effect of inspection errors we will again use Guenther's procedure. Following are the guide lines to design the double sampling plan based upon Guenther's procedure [Guenther (36)].

1. For a given (AQL, α), (RQL, β) find a single sampling plan (n, c).
2. Select any value of $c_2 \geq c$, where c is the acceptance number for single sampling plan found in Step 1.
3. Select any value of c_1, such that $0 \leq c_1 \leq c$.

4. For the c_1 chosen in Step 3, find n_1 such that probability of acceptance at AQL is less than but nearly $1 - \alpha$. (Note that the phrase 'less than but nearly' will bring subjectivity in the design procedure).

5. Check if for n_1 and c_1 found in Step 4, probability of acceptance is less than but nearly β. If not increase c_1 by 1 and go to Step 4.

6. For the set of n_1 and c_1 obtained in Step 6, choose c_2 such that $c_2 > c_1 + 1$.

7. For the value of c_2, find n_2 such that probability of acceptance (on second sample n_2 provided one is taken) at AQL satisfies the difference between $1-\alpha$ and the probability of acceptance on first sample, which was left uncovered in Step 4.

8. For the set n_2 and c_2, check if the probability of acceptance on the second sample at RQL satisfies the difference between β and the probability of acceptance on first sample which was left uncovered in step 5. If difference left uncovered is satisfied stop, else increase c_2 by 1 and go to Step 7.

The probability of taking second sample, if the number of defectives found in the first sample is between c_1 and c_2, $(c_1 < x_1 < c_2)$ can be easily found knowing n_1, c_1, c_2 for all suitable values of x_1 (This is required in Step 7).

Design in Presence of Errors:

Similar to the analysis done in the case of single sampling plan, AQL and RQL (will be termed as p_1 and p_2 respectively for simplicity) are modified in presence of errors as follows:

$$\begin{aligned} p_{1e} &= p_1 (1 - e_2) + (1 - p_1) e_1 \\ p_{2e} &= p_2 (1 - e_2) + (1 - p_1) e_1 \end{aligned} \qquad (3.27)$$

For a given set of (p_1, α) and (p_2, β) in presence of error pair (e_1, e_2) the equivalent set will be (p_{1e}, α), (p_{2e}, β). The Guenther's procedure is then used to find double sampling plan corresponding to equivalent set. If the plan so obtained is used with erroneous inspector, the observed OC curve will fit the required points (AQL, α), (RQL, β) or (p_1, α), (p_2, β). The rest of the observed OC curve (OC curve with erroneous inspector) may still fall above, below or may cross the desired OC curve at some third point between p_1 and p_2.

Table 19 shows Double sampling plans determined by the above procedure for few sets of (AQL, α), (RQL, β) requirements and various error-pairs (e_1, e_2). A notable observation is that the type I error significantly affects the plan. It changes n_1, n_2, c_1, c_2 the effect of type II errors is, however, just a slight increase in n_1 and/or n_2 keeping c_1 and c_2 unaffected, for a practical range of incoming quality and reasonable requirements on the plan.

TABLE 19: DOUBLE SAMPLING PLANS IN PRESENCE OF ERRORS FOR VARIOUS VALUES OF AQL,RQL

ALPHA=0.05,BETA=0.10 FOR ALL AQL,RQL REQUIREMENTS

ERROR-PAIRS	AQL/RQL					
(e1,e2)	0.01/0.08	0.01/0.10	0.01/0.12	0.01/0.16	0.02/0.16	0.04/0.16
(0.000,0.000)	(48,1,63,3)	(39,1,40,3)	(32,1,35,3)	(24,1,23,3)	(24,1,23,3)	(41,3,22,5)
(0.000,0.050)	(51,1,61,3)	(41,1,43,3)	(34,1,35,3)	(25,1,27,3)	(25,1,27,3)	(43,3,26,5)
(0.000,0.075)	(52,1,66,3)	(42,1,45,3)	(35,1,36,3)	(26,1,26,3)	(26,1,26,3)	(44,3,28,5)
(0.000,0.100)	(54,1,63,3)	(43,1,49,3)	(36,1,37,3)	(26,1,34,3)	(26,1,34,3)	(46,3,24,5)
(0.000,0.125)	(55,1,74,3)	(41,1,51,3)	(37,1,39,3)	(27,1,32,3)	(27,1,32,3)	(47,3,27,5)
(0.000,0.150)	(57,1,70,3)	(46,1,54,3)	(38,1,41,3)	(28,1,32,3)	(28,1,32,3)	(48,3,32,5)
(0.000,0.200)	(61,1,71,3)	(48,1,63,3)	(40,1,49,3)	(30,1,32,3)	(30,1,32,3)	(51,3,35,5)
(0.010,0.000)	(59,2,73,5)	(48,2,41,4)	(30,1,31,3)	(22,1,29,3)	(31,2,22,4)	(39,3,31,6)
(0.025,0.000)	(89,5,56,8)	(51,3,40,6)	(46,3,29,5)	(23,2,16,4)	(36,3,20,5)	(50,5,29,8)
(0.050,0.000)	(111,9,115,18)	(71,6,23,13)	(55,5,46,9)	(32,3,27,6)	(38,4,37,6)	(57,7,45,13)
(0.075,0.000)	(140,16,121,29)	(97,11,84,20)	(68,8,61,15)	(40,5,31,9)	(51,7,31,11)	(67,10,61,19)
(0.100,0.000)	(141,24,109,46)	(110,16,95,29)	(84,12,54,20)	(52,8,24,12)	(56,9,52,17)	(80,14,79,26)
(0.010,0.100)	(65,2,80,5)	(53,2,30,4)	(33,1,36,3)	(25,1,27,3)	(34,2,27,4)	(51,4,29,6)
(0.025,0.100)	(97,5,76,9)	(70,4,54,7)	(51,3,27,5)	(31,2,24,4)	(40,3,20,5)	(55,5,42,9)
(0.025,0.200)	(120,6,83,10)	(77,4,75,8)	(56,3,49,6)	(35,2,26,4)	(44,3,38,6)	(61,5,58,10)
(0.050,0.200)	(171,13,135,23)	(112,9,93,16)	(83,7,48,11)	(45,4,41,8)	(53,5,61,11)	(82,9,64,16)
(0.100,0.100)	(221,29,205,55)	(148,20,113,35)	(101,14,77,25)	(61,9,37,15)	(71,11,50,19)	(96,16,85,30)
(0.125,0.125)	(PLAN NOT FOUND)	(183,29,161,54)	(128,21,122,40)	(79,12,78,25)	(89,16,64,27)	(121,23,110,43)

PLAN: (n1,c1,n2,c2)

The design procedure by Guenther (36) is not very effective. It does not give the plan which satisfies the requirements to a close approximation. A better way would be to have an adaptive procedure but the effort required in that case would be much more. Moreover, for purpose of comparison Guenther's procedure can be used.

3.4 Analysis of Double Sampling Plan Based on AOQL and ATI Requirements:

It was shown in Sec. 3.2.3, how AOQ varies in the presence of errors. For a double sampling plan in presence of type I error AOQ decreases for all incoming quality levels. However, a plan designed on the basis of AOQL and minimum ATI with assumption of perfect inspection will not be satisfactory as ATI_e will be much higher in presence of type I errors. Similarly with type II errors the ATI_e would be less than ATI but AOQ_e will be more than AOQ. The shape of AOQ_e is also significantly different in presence of type II errors. However, one can design the plan thatwould limit the AOQ to certain AOQL provided incoming quality is not worse than say p_x. Then plan design would also include determination of this point p_x. In most of the practical situations both types of error will be present and the curve may not show any decline for the whole range of incoming quality (Sec. 3.2.3, Fig. 7).

Thus in the presence of inspection errors AOQL based design is recommended if 1) only type I errors are present,

2) only type II errors are present, 3) both type I and type II errors are present but AOQ curve is not continuously rising one. The condition 3) can be ascertained for a plan designed with the assumption of error free inspection. In all three cases suitable heuristic methods can be developed to design a suitable plan.

3.5 Economic Analysis of a Double Sampling Plan:

In the previous section we studied the effects of inspection errors on statistical measures of a double sampling. Let us now attempt to analyze the effect of inspection errors on the economics of a double sampling plan. A double sampling plan is characterized by four numbers n_1, c_1, n_2 and c_2. Let the size of the lot be denoted by N. The plan proceeds as follows.

From the lot of size N take a sample of size n_1. Let x_1 be number of defectives found in n_1. Then if

$x_1 \leq c_1$ accept the lot

$x_1 \geq c_2$ reject the lot

$c_1 < x_1 < c_2$ take another sample of size n_2.

Let x_2 be the number of defectives found in the second sample then if,

$x_1 + x_2 \leq c_2$ accept the lot

otherwise reject

Further let c_1, c_r and c_a be cost of inspection, cost of repair and cost of acceptance of a defective, described

as in Chapter II. Usually in case of sampling plans, we assume Binomial distribution for number of defectives in the sample along with Binomial sampling. For the case of economic analysis Binomial distribution of defectives is not suitable because in that case we have incoming quality p as a parameter. We will assume p, the probability of obtaining a defective as Beta distributed for Beta assumes various shapes depending upon the parameters.

Model Development Without Errors:

In absence of errors the various costs are as follows. The cost of acceptance on the first sample Ca_1 is given by,

$$Ca_1 = c_i n_1 + c_r x_1 + c_a (X - x_1) \quad \text{if } x_1 \leq c_1$$

where,

X - Number of the defectives in the lot of size N.

$c_i n_1$- Cost of inspecting n_1 items

$c_r x_1$- Cost of repair of x_1 defectives found

$X - x_1$- Number of defectives going in the uninspected portion

$c_a(X-x_1)$- Cost of accepting the defectives.

Let, $X - x_1 = u_1$

$$Ca_1 = c_i n_1 + c_r x_1 + c_a u_1 \qquad x_1 \leq c_1 \tag{3.28}$$

Similarly cost of rejection on the first sample, Cr_1 is given by,

$$Cr_1 = c_i N + c_r x_1 + c_r u_1 \qquad x_1 \geq c_2 \tag{3.29}$$

We go to second sample if $c_1 < x_1 < c_2$.

Cost of acceptance Ca_2 and cost of rejection Cr_2 on the second sample are given by,

$$Ca_2 = c_i n_i + c_i n_2 + c_r x_1 + c_r x_2 + c_a (u_1 - x_2).$$

Let $u_1 - x_2 = u_2$,

$$Ca_2 = c_i n_1 + c_i n_2 + c_r x_1 + c_r x_2 + c_a u_2 \quad (3.30)$$

$$Cr_2 = c_i N + c_r x_1 + c_r x_2 + c_r u_2 \quad (3.31)$$

Then based on the above cost terms the expected total cost E(TC) is given by,

$$\begin{aligned} E(TC) = & \sum_{x_1=0}^{c_1} (K_{11} + C_{11} E(u_1/x_1)) g_{n_1}(x_1) \\ & + \sum_{x_1=c_2}^{n_1} (K_{12} + C_{12} E(u_1/x_1)) g_{n_1}(x_1) \\ & + \sum_{x_1=c_1+1}^{c_2-1} [\sum_{x_2=0}^{c_2-x_1} (K_{21} + C_{21} E(u_2/x_2, A)) g_{n_2}(x_2/A) \\ & + \sum_{x_2=c_2-x_1+1}^{n_2} (K_{22} + C_{22} E(u_2/x_2, A)) g_{n_2}(x_2/A)].g_{n_1}(x_1) \end{aligned} \quad (3.32)$$

where,

$$K_{11} = c_i n_1 + c_r x_1$$

$$C_{11} = c_a$$

$$K_{12} = c_i N + c_r x_1$$

$$C_{12} = c_r$$

$$K_{21} = c_i n_1 + c_i n_2 + c_r x_1 + c_r x_2$$

$$C_{21} = c_a$$

$$K_{22} = c_i N + c_r x_1 + c_r x_2$$

$$C_{22} = c_r$$

$g_{n_1}(x_1)$ = Marginal distribution of the number of defectives in the first sample of size n_1

$E(u_1/x_1)$ = Conditional expectation of number of defectives in the remaining portion of lot $(N - n_1)$ after first sample has been drawn, and it contains x_1 defectives.

A = Event when $c_1 < x_1 < c_2$

$g_{n_2}(x_2/A)$ = Conditional distribution of number of defectives in the second sample of size n_2 given that event A has occurred.

$E(u_2/x_2, A)$ = Conditional expectation of number of defectives remaining in the uninspected portion of lot after the second sample has been drawn $(N - n_1 - n_2)$ given that x_2 defectives are found in the second sample of size n_2 and event A has occurred.

Distributional Considerations:

Let $f_N(X)$ be the distribution of the number of defectives X in the lot of size N. If x_1 is the number of defectives

in the sample of size n_1, the conditional distribution $f_{n_1}(x_1/X)$ is hypergeometric,

$$f_{n_1}(x_1/X) = \frac{\binom{n_1}{x_1}\binom{N-n_1}{X-x_1}}{\binom{N}{X}} \tag{3.33}$$

Then by Bayes theorem the marginal distribution $g_{n_1}(x_1)$ of the number of defectives x_1 in the first sample is given by,

$$g_{n_1}(x_1) = \sum_{X=x_1}^{N-n_1+x_1} f_{n_1}(x_1/X).f_N(X)$$

Substituting $f_{n_1}(x_1/X)$ and also $u_1 = X-x_1$, we get,

$$g_{n_1}(x_1) = \sum_{u_1=0}^{N-n_1} \frac{\binom{n_1}{x_1}\binom{N-n_1}{u_1}}{\binom{N}{x_1+u_1}} \cdot f_N(x_1+u_1) \tag{3.34}$$

Then the conditional distribution $h(u_1/x_1)$ of number of defectives u_1 in the remaining lot $(N - n_1)$ after the first sample is drawn and x_1 defectives are found is given by,

$$h(u_1/x_1) = \frac{\binom{n_1}{x_1}\binom{N-n_1}{u_1}}{\binom{N}{x_1+u_1}} \frac{f_N(x_1+u_1)}{g_{n_1}(x_1)} \tag{3.35}$$

Conditional expectation of the number of defectives u_1 will be then given by,

$$E(u_1/x_1) = \sum_{u_1=0}^{N-n_1} u_1\, h(u_1/x_1)$$

The above equation can be simplified as shown by Worthram et al (28) to the form,

$$E(u_1/x_1) = (N - n_1) \frac{(x_1+1)}{(n_1+1)} \frac{g_{n_1+1}(x_1+1)}{g_{n_1}(x_1)} \qquad (3.36)$$

We go to the second sample if x_1 is such that $c_1 < x_1 < c_2$. Let A be the event when $c_1 < x_1 < c_2$. All of the following expressions will be valid only if event A occurs. The conditional distribution of x_2 defectives found in the second sample of size n_2, given that A has occurred is as follows.

$$g_{n_2}(x_2/A) = \sum_{u_1=x_2}^{N-n_1-n_2+x_2} \frac{\binom{n_2}{x_2}\binom{N-n_1-n_2}{u_1-x_2}}{\binom{N-n_1}{u_1}} f_{N-n_1}(X-x_1/A)$$

We can substitute,

$$f_{N-n_1}(X-x_1/A) = h(u_1/x_1)$$

$$u_2 = u_1 - x_2$$

$$g_{n_2}(x_2/A) = \sum_{u_2=0}^{N-n_1-n_2} \frac{\binom{n_2}{x_2}\binom{N-n_1-n_2}{u_2}}{\binom{N-n_1}{u_2+x_2}} h(u_2+x_2/x_1) \qquad (3.37)$$

Then conditional distribution $\ell(u_2/x_2, A)$ of the number of defectives u_2 remaining in the uninspected portion of the lot $(N - n_1 - n_2)$ after the second sample has been drawn and x_2 defectives have been found is given by,

$$\ell(u_2/x_2, A) = \frac{\binom{n_2}{x_2}\binom{N-n_1-n_2}{u_2}}{\binom{N-n_1}{u_2+x_2}} \frac{h(u_2+x_2/A)}{g_{n_2}(x_2/A)} \qquad (3.38)$$

Then conditional expectation of u_2 given that A has occurred and second sample of size n_2 contains x_2 defectives is given by,

$$E(u_2/x_2, A) = \sum_{u_2=0}^{N-n_1-n_2} u_2 \; h(u_2/x_2, A) \tag{3.39}$$

Thus knowing all the distributions and expectations involved along with the cost coefficients, the total expected cost E(TC) can be found for any given plan (N, n_1, c_1, n_2, c_2). However, the design of plan still remains an open one. One can try to develop a heuristic based on the above analysis. Some subjective information such as, n_1 and n_2 should be equal etc. can be provided and based on this information heuristic might give a plan with better economics.

<u>A Particular Case:</u>

Let us now consider a particular case of the distribution of the p, p being the probability of getting a defective. Let p be Beta distributed with parameters s and t $\beta(s, t)$,

$$f(p) = \beta(s, t) = \frac{\Gamma(s+t)}{\Gamma(s)\,\Gamma(t)} \; p^{s-1} \; (1-p)^{t-1} \tag{3.40}$$

$$t, s > 0, \quad 0 \leq p \leq 1$$

Then distribution $f_N(X)$ of the number of defectives X in the lot of size N is given by,

$$f_N(X) = \int_0^1 \binom{N}{X} p^X \; (1-p)^{N-X} \; f(p) \; dp \tag{3.41}$$

$$= \int_0^1 \binom{N}{X} p^X \; (1-p)^{N-X} \frac{\Gamma(s+t)}{\Gamma(s)\,\Gamma(t)} \; p^{s-1} \; (1-p)^{t-1} \; dp$$

which on simplification gives,

$$f_N(X) = \binom{N}{X} \frac{\Gamma(s + X)}{\Gamma(s)} \frac{\Gamma(s + t)}{\Gamma(t)} \frac{\Gamma(t + N - X)}{\Gamma(s + t + N)} \qquad (3.42)$$

i.e. the number of defectives X, in the lot of size N, follow a Polya distribution.

Hald (19) has showed that compound hypergeometric distribution given in Eq. (3.34) is of the same form as the prior $f_N(X)$ in case of reproducible prior distributions such as Binomial, Mixed Binomial, Polya, Rectangular etc. with N and X replaced by n_1 and x_1. Hence we get,

$$g_{n_1}(x_1) = \binom{n_1}{x_1} \frac{\Gamma(s + x_1)}{\Gamma(s)} \frac{\Gamma(s + t)}{\Gamma(t)} \frac{\Gamma(t + n_1 - x_1)}{\Gamma(s + t + n_1)} \qquad (3.43)$$

For the case when we have 'p' as Beta distributed we have seen $f_N(X)$, $g_{n_1}(x_1)$ are Polya. Further from Eq. (3.36),

$$E(u_1/x_1) = (N - n_1) \frac{(x_1 + 1)}{(n_1 + 1)} \frac{g_{n_1+1}(x_1+1)}{g_{n_1}(x_1)}$$

Substituting the values of $g_{n_1+1}(x_1+1)$ and $g_{n_1}(x_1)$, we can simply Eq. (3.36) to get,

$$E(u_1/x_1) = (N - n_1) \frac{(s + x_1)}{(s + t + n_1)} \qquad (3.44)$$

The conditional distribution of u_1, $h(u_1/x_1)$ is given by Eq. (3.35) as,

$$h(u_1/x_1) = \frac{\binom{n_1}{x_1}\binom{N - n_1}{u_1}}{\binom{N}{x_1+u_1}} \frac{f_N(x_1+u_1)}{g_{n_1}(x_1)}$$

Substituting for $f_N(x_1 + u_1)$, $g_{n_1}(x_1)$ from Eq. (3.42), (3.43) respectively, we get,

$$h(u_1/x_1) = \frac{\binom{n_1}{x_1}\binom{N-n_1}{u_1}}{\binom{N}{x_1+u_1}} \quad \frac{\binom{N}{x_1+u_1}\dfrac{\Gamma(s+x_1+u_1)}{\Gamma(s)}\,\dfrac{\Gamma(s+t)}{\Gamma(t)}\,\dfrac{\Gamma(t+N-x_1-u_1)}{\Gamma(s+t+N)}}{\binom{n_1}{x_1}\dfrac{\Gamma(s+x_1)}{\Gamma(s)}\,\dfrac{\Gamma(s+t)}{\Gamma(t)}\,\dfrac{\Gamma(t+n_1-x_1)}{\Gamma(s+t+u_1)}}$$

$$= \binom{N-n_1}{u_1}\frac{\Gamma(s+x_1+u_1)}{\Gamma(s+x_1)}\,\frac{\Gamma(s+t+n_1)}{\Gamma(s+t+N)}\,\frac{\Gamma(t+N-x_1-u_1)}{\Gamma(t+n_1-x_1)} \tag{3.45}$$

Let,

$$N_1 = N - n_1$$

$$s_1 = s + x_1$$

$$t_1 = t + n_1 - x_1$$

for a given n_1, x_1, s_1 and t_1 are constants,

$$h(u_1/x_1) = \binom{N_1}{u_1}\frac{\Gamma(s_1+u_1)}{\Gamma(s_1)}\,\frac{\Gamma(s_1+t_1)}{\Gamma(t_1)}\,\frac{\Gamma(t_1+N_1-u_1)}{\Gamma(s_1+t_1+N_1)} \tag{3.46}$$

which is again Polya with s_1, t_1 as parameters and N and X being replaced by N_1 and u_1 respectively. Then again by the property of reproducibility distribution of x_2 given A has occurred is given by,

$$g_{n_2}(x_2/A) = \binom{n_2}{x_2}\frac{\Gamma(s_1+x_2)}{\Gamma(s_1)}\,\frac{\Gamma(s_1+t_1)}{\Gamma(t_1)}\,\frac{\Gamma(t_1+n_2-x_2)}{\Gamma(s_1+t_1+n_2)} \tag{3.47}$$

One may note that Eq. (3.46) and Eq. (3.47) are not independent

of x_1, parameters s_1 and t_1 are functions of x_1. Further, we have expectation of u_2 given that A has occurred and x_2 defectives have been found in the second sample of size n_2 from Eq. (3.39) as follows,

$$E(u_2/x_2, A) \quad = \quad \sum_{u_2=0}^{N-n_1-n_2} u_2 \; \ell(u_2/x_2, A)$$

The above equation can be simplified on the same lines as before, to get,

$$E(u_2/x_2, A) \quad = \quad (N_1 - n_2) \; \frac{(s_1 + x_2)}{(s_1 + t_1 + n_2)} \qquad (3.48)$$

The above relations are developed when probability of getting a defective follows a Beta distribution (A general case). These simplified relationships can be used to evaluate total expected cost E(TC) from Eq. (3.32).

Model Development in Presence of Errors:

First Sample:

Let us first try to develop expressions for the first sample. These can be used with simple modifications in case of single sampling plan. First a simple of size n_1 is drawn. Let,

x_1 = number of defectives present

x_{1e} = number of observed defectives

i_1 = number of defectives (out of x_1) classified correctly

i_2 = number of good items (out of $n_1 - x_1$) classified erroneously as defectives

u_1 = number of defectives remaining in the lot after first sample has been drawn

x_{1e} = $i_1 + i_2$

$c_i n_1$ = cost of inspection of n_1 items

$c_r(i_1+i_2)$ = Cost of repair of $i_1 + i_2$ items, observed to be defectives

$c_a(x_1-i_1)$ = cost of accepting $x_1 - i_1$ defective items classifying them as good.

Then distribution of defectives classified correctly (i_1) is given by [Collins et al (17)],

$$f_1(i_1) = \binom{x_1}{i_1} (1 - e_2)^i \; e_2^{\,x_1-i} \tag{3.49}$$

Distribution $f_2(i_2)$ of good items classified erroneously as good (i_2) is given by

$$f_2(i_2) = \binom{n_1-x_1}{i_2} e_1^{\,i_2} (1 - e_1)^{n_1-x_1-i_2} \tag{3.50}$$

Cost of Acceptance:

Acceptance is possible on first sample if $x_{1e} \leq c_1$ or $i_1 + i_2 \leq c_1$. Expected cost of acceptance on the first sample $E(Ca_1)$ can be given by,

$$E(Ca_1) = \sum_{x_1=0}^{n_1} \Big[\sum_{i_1+i_2=0}^{c_1} \{c_1 n_1 + c_r(i_1+i_2) + c_a(x_1-i_1) + \sum_{u_1=0}^{N-n_1} c_a u_1 h(u_1/x_1)\} f_1(i_1).f_2(i_2)\Big] g_{n_1}(x_1) \tag{3.51}$$

where, $\sum_{u_1=0}^{N-n_1} u_1 \; h(u_1/x_i)$ is given by Eq. (3.35).

The above term is multiplied by c_a i.e. cost of accepting a defective. Further, it is multiplied by

$$\sum_{i_1+i_2=0}^{c_1} f_1(i_1) \; f_2(i_2)$$

which is probability of acceptance on first sample for given x_1. Finally multiplied by $g_{n_1}(x_1)$ and summed over all possible values of x_1. Similar is the explanation for other terms.

Cost of Rejection:

When decision to reject the lot on the basis of first sample is taken, the remaining portion of lot $(N - n_1)$ is inspected 100 percent.

In that case u_1, the number of defectives present in remaining lot will be subjected to erroneous classification. Let,

j_1 = Number of defectives (out of u_1) classified correctly.

j_2 = Number of good items (out of $N-n_1-u_1$) classified erroneously as bad.

$$f_3(j_1) = \binom{u_1}{j_1} (1 - e_2)^{j_1} \; e_2^{\,u_1-j_1} \tag{3.52}$$

$$f_4(j_2) = \binom{N-n_1-u_1}{j_2} \; e_1^{\,j_2} \; (1-e_1)^{N-n_1-u_1-j_2} \tag{3.53}$$

The decision to reject the lot on the basis of first sample is taken if $c_2 \leq (i_1 + i_2) \leq n_1$. Then expected cost of rejection on the first sample $E(Cr_1)$ can be given by,

$$E(Cr_1) = \sum_{x_1=0}^{n_1} \Big[\sum_{i_1+i_2=c_2}^{n_1} \{c_1 N + c_r (i_1+i_2) + c_a(x_1-i_1)$$
$$+ \sum_{u_1=0}^{N-n_1} \{ \sum_{j_1+j_2=0}^{u_1} \{c_r(j_1+j_2) + c_a(u_1-j_1)\} f_3(j_1) f_4(j_2)\}$$
$$\cdot\ h(u_1/x_1)\}\ f_1(i_1)\ f_2(i_2)\Big]\ g_{n_1}(x_1) \qquad (3.54)$$

<u>Second Sample</u>:

The second sample is drawn when decision is not possible on the first sample. This happens when the number of observed defectives is such that $c_1 < x_{1e} < c_2$ i.e. $c_1 < i_1 + i_2 < c_2$. The second sample will lead to acceptance of lot if cummulative number of observed defectives is less than or equal to c_2 else it will be rejected. Rest of the analysis being on the same lines we write expression for the expected cost of acceptance on the second sample as follows:

$$E(Ca_2) = \sum_{x_1=0}^{n_1} \Big[\sum_{i_1+i_2=c_1+1}^{c_2-1} \Big[\sum_{x_2=0}^{n_2} \Big[\sum_{k_1+k_2=0}^{c_2-(i_1+i_2)} [(n_1+n_2)c_i$$
$$+ c_r(k_1+k_2) + c_r(i_1+i_2) + c_a(x_1-i_1) + c_a(x_2-k_1)$$
$$+ \sum_{u_2=0}^{N-n_1-n_2} c_a\, u_2\, \ell(u_2/x_2, A)]\ f_5(k_1) f_6(k_2)\Big]$$
$$\cdot\ g_{n_2}(x_2/A)\Big]\ f_1(i_1) f_2(i_2)\Big]\ g_{n_1}(x_1) \qquad (3.55)$$

where,

k_1 = Number of defectives out of x_2 present in n_2 which are classified correctly.

k_2 = Number of good items in second sample out of n_2-x_2 classified erroneously as defectives.

$$f_5(k_1) = \binom{x_2}{k_1} (1-e_2)^{k_1} e_2^{x_2-k_1} \qquad (3.56)$$

$$f_6(k_2) = \binom{n_2-x_2}{k_2} e_1^{k_2} (1-e_1)^{n_2-x_2-k_2} \qquad (3.57)$$

In the same way expected cost of rejection $E(Cr_2)$ on the second sample can be given by,

$$E(Cr_2) = \sum_{x_1=0}^{n_1} \Big[\sum_{i_1+i_2=c_1+1}^{c_2-1} \Big[\sum_{x_2=0}^{n_2} \Big[\sum_{k_1+k_2=c_2+1-i_1-i_2}^{n_2} \sum^{n_2} [c_i N$$

$$+ c_r(i_1+i_2) + c_r(k_1+k_2) + c_a(x_1-i_1) + c_a(x_2-k_1)$$

$$+ \sum_{u_2=0}^{N-n_1-n_2} \Big[\sum_{\ell_1+\ell_2=0}^{n_2} [c_r(\ell_1+\ell_2)+c_a(u_2-\ell_1)]\, f_7(\ell_1) f_8(\ell_2)]$$

$$\cdot\ \ell(u_2/x_2, A)]\, f_5(k_1)\, f_6(k_2)]\, g_{n_2}(x_2/A)]$$

$$\cdot\ f_1(i_1)\, f_2(i_2)]\, g_{n_1}(x_1) \qquad (3.58)$$

where,

ℓ_1 = Number of defectives present(u_2) in the rejected portion of lot after second sample has been drawn, which are classified correctly.

ℓ_2 = Number of good items present $(N-n_1-n_2-u_2)$ in rejected portion of lot after second sample (out of $N-n_1-n_2-u_2$) classified erroneously as defectives.

$$f_7(\ell_1) = \binom{u_2}{\ell_1} (1-e_2)^{\ell_1} e_2^{u_2-\ell_1} \quad (3.59)$$

$$f_8(\ell_2) = \binom{N-n_1-n_2-u_2}{\ell_2} e_1^{\ell_2} (1-e_1)^{N-n_1-n_2-u_2-\ell_2} \quad (3.60)$$

In the above equations from (3.49) to (3.60) we have discussed the expected costs of acceptance and rejection for the first sample and the second sample. Therefore expected total cost E(TC) can be given as,

$$E(TC) = E(Ca_1) + E(Cr_1) + E(Ca_2) + E(Cr_2) \quad (3.61)$$

In the Eq. (3.51) for $E(Ca_1)$ we have a summation over two variables, $\sum_{i_1+i_2=0}^{c_1}$. This summation has to be separated into double summation over each of the two independent variables i_1 and i_2. The double summation is given by,

$$\sum_{i_2=0}^{c_1} \sum_{i_1=0}^{c_1-i_2} \quad (3.62)$$

The distribution of i_1 is given by Eq. (3.49) (which is Binomial). The range of i_1 is therefore given by $0 \leq i_1 \leq x_1$. Similarly distribution of i_2 is given by Eq. (3.50), which is again Binomial. Range of i_2 is then given by $0 \leq i_2 \leq n_1-x_1$. The double summation given above is modified to,

$$\sum_{i_2=0}^{\mathrm{Min}[c_1,(n_1-x_1)]} \quad \sum_{i_1=0}^{\mathrm{Min}[(c_1-i_2),x_1]} \tag{3.63}$$

The above double summation can be used in the Eq. (3.51). In the same way summation over two variables i_1+i_2 with other ranges, j_1+j_2 etc. occur in Eqs.(3.54), (3.55) and (3.58). These can be separated into the double summation over two variables on the same lines as discussed above.

Eq. (3.61) is the expected total cost of a double sampling plan. The various distributions used have already been discussed for the case when model was developed in the absence of errors. This equation can be used to analyze the effect of errors on the economics of a double sampling plan. But that is not done due to shortage of time. However, the exact economic analysis of the double sampling plan in the presence of inspection errors gives us a better insight to claim that the model developed for the single sampling plan in presence of errors by Case et al (27) requires certain changes. However, that can be used as an approximate model to show the adverse effects of errors on single sampling plan as discussed in Chapter II.

3.6 Conclusions:

Let us finally summarize the results obtained in this chapter.

1. The effect of errors on the incoming quality does not depend upon the plan.

2. Probability of acceptance increases in presence of type II error and decreases in presence of type I errors.

3. For a given sampling plan the average outgoing quality decreases in the presence of type I errors because of more screening inspection. The average outgoing quality increases in presence of type II errors. The shape also is significantly different. The effect is similar to that in case of single sampling plan.

4. Average sample number depends upon the probability of acceptance and probability of rejection on the first sample and hence the net effect may be that ASN_e is greater or less than ASN.

5. Average total inspection increases in the presence of type I errors and decreases in the presence of type II errors.

6. Design of double sampling plan based on (AQL, α), (RQL, β) requirements was considered in presence of errors. Type II error increases n_1 and/or slightly, and does not affect c_1 and c_2. Type I error increases n_1, n_2, c_1 and c_2 appreciably.

7. The economic analysis of a double sampling plan in presence of errors is presented in form of a model. The computational results have not been presented. A effort in that direction may be fruitful in recognizing adverse cost effects due to presence of inspection errors.

CHAPTER IV

INSPECTION ERRORS AND SEQUENTIAL SAMPLING PLAN

4.1 Introduction:

A single sampling plan has a fixed sample size. Double sampling and multiple sampling do not have a fixed sample size, the sample size instead depends upon the quality of the items being inspected. In sequential sampling the size of the sample for a particular lot is left undetermined. In sequential sampling we inspect one or several pieces at a time until the cumulated evidence is strong enough one way or the other to call the lot acceptable or not.

The Sequential Method of Acceptance Sampling:

A sequential inspection plan consists of examining in sequence single items (or group of items) chosen at random from a lot and at each step making one of the following three decisions.

(i) The lot is acceptable

(ii) The lot is unacceptable

(iii) The evidence is insufficient for arriving at either of the two decisions without too great a risk of error.

When the third course of action is taken, an additional item (or group of items) is inspected and same three decisions are considered again. The inspection is continued until the decision of acceptance or rejection is arrived at.

The criterion for each of the actions is conveniently given by a pair of parallel straight lines, which depend upon the quality tolerance limits and preassigned risks. The two lines are plotted on a chart as shown below:

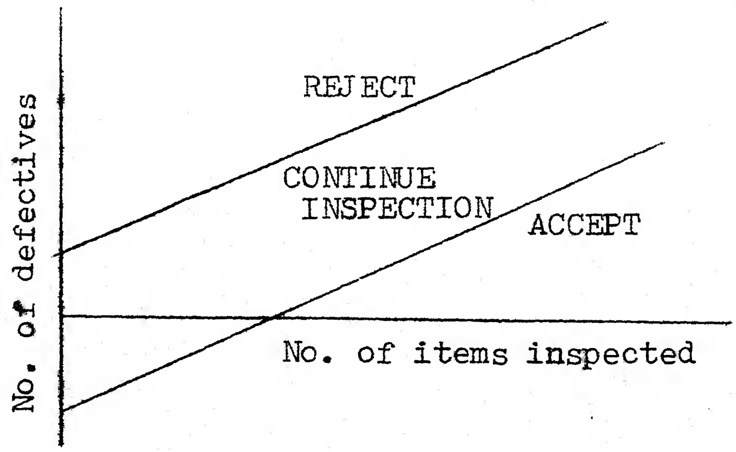

Fig. 10: Graphical form of the sequential sampling plan.

Inspector plots the number of defectives observed against total number of items inspected. As long as the plotted points fall between the two lines the inspector continues to draw additional items. Inspection terminates when a plotted point falls on/above the upper line or when a plotted point falls on/below the lower line, the lot being accepted in the latter case and rejected if its the former.

The procedure need not be carried out graphically. A table can be easily prepared giving the acceptance and rejection numbers for each of the number of items that could be inspected. The tabular form of the sequential sampling plan is shown below:

n	d_1 Acceptance number	d Number of defectives observed	d_2 Rejection number
1	.	.	.
2	.	.	.
.			
.			
.			

Tabular form of sequential sampling plan.

In this chapter, we will study the effect of inspection errors on this very useful acceptance sampling plan. We will first consider the effect of inspection errors on various performance measures. We will then attempt to design a sequential sampling plan based upon (AQL, α), (RQL, β) requirements when inspection errors are present. Finally, we will compare three plans: single, double and sequential sampling plans designed to satisfy the same (AQL, α), (RQL, β) requirement. The comparison of the three plans will be on the basis of effect of inspection errors on the probability of acceptance.

4.2 Effect of Inspection Errors on Performance Measures:

4.2.1 Apparent Incoming Quality:

Apparent incoming quality for a sequential sampling plan will be same as in case of single or double sampling plans. Apparent incoming quality is given by,

$$p_e = p(1 - e_2) + (1 - p)\, e_1 \tag{4.1}$$

4.2.2 Probability of Acceptance:

Let us start with the assumption that size of the lot submitted for inspection is large relative to the number of items inspected so that the probability of getting any number of defectives (say r) is given by a Binomial mass function.

Analysis in Absence of Error:

It is known that a discrete variable, the number of successes r follows a Binomial distribution whose parameter p (the fraction defective) is unknown. We wish to decide between the two hypotheses $p = p_1$ (AQL) and $p = p_2$ (RQL) with risks of error limited to small quantities α and β. The actions are related to likelihood ratio L as given below [SRG report (13)].

$$L = \frac{\binom{n}{r} p_2^{\,r} (1-p_2)^{n-r}}{\binom{n}{r} p_1^{\,r} (1-p_1)^{n-r}} \begin{cases} \geq A & \text{accept } p = p_2 \\ \leq B & \text{accept } p = p_1 \\ < A \text{ and } > B & \text{reserve judgement and take additional observation.} \end{cases} \tag{4.2}$$

Taking logarithms Eq. (4.2) becomes,

$$r \log \frac{p_2}{p_1} - (n-r) \log \frac{(1-p_1)}{(1-p_2)} \begin{cases} \geq a \\ \leq -b \\ < a \text{ and } > -b \end{cases} \qquad (4.3)$$

We also have following relations because of specified risk probabilities [SRG report (12)]

$$\begin{aligned} 1 - \beta &\geq A\,\alpha \\ \beta &\leq B\,(1 - \alpha) \end{aligned} \qquad (4.4)$$

But we actually work with

$$\begin{aligned} A &= \frac{1 - \beta}{\alpha} \\ B &= \frac{\beta}{(1 - \alpha)} \end{aligned} \qquad (4.5)$$

$$r = \frac{\log \dfrac{(1 - p_1)}{(1 - p_2)}}{\log \dfrac{p_2}{p_1} \dfrac{(1 - p_1)}{(1 - p_2)}}\, n - \frac{b}{\log \dfrac{p_2}{p_1} \dfrac{(1 - p_1)}{(1 - p_2)}} \qquad (4.6)$$

Acceptance line

$$r = \frac{\log \dfrac{(1 - p_1)}{(1 - p_2)}}{\log \dfrac{p_2}{p_1} \dfrac{(1 - p_1)}{(1 - p_2)}}\, n + \frac{a}{\log \dfrac{p_2}{p_1} \dfrac{(1 - p_1)}{(1 - p_2)}} \qquad (4.7)$$

Rejection line

where,

$$\begin{aligned} a &= \log A = \log \frac{1 - \beta}{\alpha} \\ b &= \log B = \log \frac{\beta}{1 - \alpha} \end{aligned} \qquad (4.8)$$

The above equations give acceptance and rejection lines as,

$$d_1 = -h_1 + s\,n \qquad \text{acceptance line} \qquad (4.9)$$

$$d_2 = h_2 + s\,n \qquad \text{rejection line} \qquad (4.10)$$

where,

$$h_1 = \frac{b}{\log \dfrac{p_2\ (1 - p_1)}{p_1\ (1 - p_2)}} \qquad (4.11)$$

$$h_2 = \frac{a}{\log \dfrac{p_2\ (1 - p_1)}{p_1\ (1 - p_2)}} \qquad (4.12)$$

and

$$s = \frac{\log \dfrac{(1 - p_1)}{(1 - p_2)}}{\log \dfrac{p_2\ (1 - p_1)}{p_1\ (1 - p_2)}} \qquad (4.13)$$

The assumption that the number of defectives in a sample of size n follow a Binomial distribution enables us to replace the incoming quality p by apparent incoming quality p_e in the above relations.

The probability of acceptance Pa cannot be expressed in terms of p. But Pa can be expressed as a function of a dummy variable w which in turn is a function of incoming quality p. These relationships are given below [SRG report (13)],

$$p' = \frac{w^s - 1}{w - 1}$$

$$Pa' = \frac{w^{h_1+h_2} - w^{h_1}}{w^{h_1+h_2} - 1} \qquad (4.14)$$

And,

$$p'' = p' \, w^{1-s}$$
$$Pa'' = w^{-h_1} \, Pa'' \qquad (4.15)$$

Eqns. (4.14) and (4.15) are valid for $w > 1$.

where h_1, h_2 and s are given by Eqs. (4.11), (4.12) and (4.13) respectively. The two sets of Eqns. (4.14) and (4.15) give two points on the OC curve for a fixed w. The Eqn. (4.14) gives a point for which $p < s$ and Eq. (4.15) gives a point for which $p > s$. For $p = s$ probability of acceptance is given by,

$$Pa = \frac{h_2}{h_1 + h_2} \qquad (4.16)$$

The above Eqns. (4.14), (4.15) and (4.16) can be modified to study the effect of inspection errors on the probability of acceptance. Inspector is provided with the values of h_1, h_2 and s, presuming that he is error free. But unfortunately he is not. The erroneous inspector observes apparent incoming quality p_e instead of true incoming quality p. Then we will have,

$$p'_e = \frac{w_e^{\,s} - 1}{w_e - 1}$$
$$Pa'_e = \frac{w_e^{\,h_1+h_2} - w_e^{\,h_1}}{w_e^{\,h_1+h_2} - 1} \qquad (4.17)$$

and,

$$p''_e = \frac{w_e - w_e^{1-s}}{w_e - 1} = p'_e \, w_e^{1-s}$$

$$Pa''_e = \frac{w_e^{h_1} - 1}{w_e^{h_1+h_2} - 1} = w_e^{-h_1} \, Pa'_e \qquad (4.18)$$

Eqns. (4.17) and (4.18) are valid $w_e > 1$.

For a particular value of the incoming quality p, we can find apparent incoming quality p_e by Eq. (4.1). If $p_e < s$, we use (4.17). If $p_e > s$, we use Eq. (4.18) [If $p_e = s$ then $Pa_e = \frac{h_2}{h_1 + h_2}$ by Eq. (4.16)]. We then solve first equation of appropriate set of equations, either Eq. (4.17) or Eq. (4.18) to obtain value of w_e. This requires some numerical method. The second equation of the same set will give Pa_e for the value of w_e we found before. Thus for a incoming quality p, Pa_e, the probability of acceptance in presence of error is obtained.

The above effort is required for directly comparing Pa_e with Pa for a particular incoming quality. To study the effect of inspection errors computations based on above analysis were done and the results are shown in the Tables 20 and 21.

Table 20 gives the probability of acceptance for few selected incoming quality levels and few representative error pairs. Table 21 gives the percent change in probability

TABLE 20: PROBABILITY OF ACCEPTENCE INPRESENCE OF ERROR(SEQUENTIAL SAMPLING PLAN)

PLAN: (h1=1.0458,h2=1.3426,s=0.0331)

(AQL=0.01, α=0.05,RQL=0.06, β=0.10)

	ERROR-PAIRS	INCOMING QUALITY 'p'							
	(e1,e2)	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
1	(0.000,0.000)	0.95000	0.81625	0.63624	0.46110	0.32013	0.21802	0.14759	0.10000
2	(0.000,0.050)	0.95463	0.83258	0.66404	0.49390	0.35165	0.24492	0.16920	0.11681
3	(0.000,0.075)	0.95687	0.84055	0.67795	0.51081	0.36833	0.25953	0.18117	0.12627
4	(0.000,0.100)	0.95905	0.84838	0.69184	0.52806	0.38564	0.27494	0.19398	0.13651
5	(0.000,0.125)	0.96118	0.85607	0.70569	0.54553	0.40358	0.29118	0.20767	0.14759
6	(0.000,0.150)	0.96325	0.86360	0.71947	0.56321	0.42215	0.30829	0.22230	0.15958
7	(0.000,0.200)	0.96722	0.87818	0.74674	0.59945	0.46110	0.34515	0.25457	0.18655
8	(0.010,0.000)	0.81790	0.63994	0.46593	0.32505	0.22230	0.15109	0.10275	0.07012
9	(0.025,0.000)	0.54995	0.39274	0.27231	0.18655	0.12750	0.08733	0.06004	0.04144
10	(0.050,0.000)	0.22230	0.15346	0.10599	0.07343	0.05106	0.03562	0.02492	0.01747
11	(0.075,0.000)	0.08164	0.05947	0.04183	0.02951	0.02086	0.01477	0.01046	0.00740
12	(0.100,0.000)	0.03304	0.02356	0.01683	0.01203	0.00860	0.00614	0.00438	0.00312
13	(0.010,0.100)	0.83419	0.67703	0.51594	0.37725	0.26971	0.19097	0.13492	0.09546
14	(0.050,0.100)	0.56774	0.42215	0.30538	0.21802	0.15497	0.11019	0.07854	0.05616
15	(0.025,0.200)	0.56574	0.45312	0.34194	0.25457	0.16838	0.13920	0.10295	0.07631
16	(0.050,0.200)	0.24023	0.17941	0.13387	0.10000	0.07186	0.05616	0.04223	0.03182
17	(0.100,0.100)	0.03431	0.02539	0.01882	0.01396	0.01036	0.00769	0.00570	0.00422
18	(0.125,0.125)	0.01377	0.01936	0.00783	0.00592	0.00447	0.00337	0.00253	0.00191

TABLE 21: PERCENTAGE CHANGE IN PROBABILITY OF ACCEPTANCE IN PRESENCE OF ERRORS (SEQUENTIAL SAMPLING PLAN)

PLAN: (h1=1.0458, h2=1.3426, s=0.0341)

(AQL=0.01, α=0.05, RQL=0.08, β=0.10)

	ERROR-PAIRS	INCOMING QUALITY 'p'							
	(e1,e2)	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	0.49	2.00	4.37	7.11	9.83	12.34	14.64	16.81
3	(0.000,0.075)	0.72	2.98	6.56	10.78	15.04	19.04	22.75	26.27
4	(0.000,0.100)	0.95	3.94	8.74	14.52	20.44	26.11	31.43	36.51
5	(0.000,0.125)	1.18	4.88	10.92	18.31	26.05	33.56	40.71	47.59
6	(0.000,0.150)	1.39	5.80	13.08	22.14	31.85	41.40	50.62	59.58
7	(0.000,0.200)	1.81	7.59	17.37	30.00	44.01	58.31	72.46	86.55
8	(0.010,0.000)	-13.91	-21.60	-26.77	-29.51	-30.57	-30.70	-30.38	-29.88
9	(0.025,0.000)	-42.11	-51.88	-57.20	-59.54	-60.18	-59.94	-59.32	-58.56
10	(0.050,0.000)	-76.60	-81.20	-83.34	-84.08	-84.05	-83.66	-83.15	-82.53
11	(0.075,0.000)	-91.07	-92.71	-93.43	-93.60	-93.48	-93.23	-92.91	-92.60
12	(0.100,0.000)	-96.52	-97.11	-97.35	-97.39	-97.31	-97.18	-97.03	-96.88
13	(0.010,0.100)	-12.19	-17.06	-18.91	-18.18	-15.76	-12.41	-8.58	-4.54
14	(0.025,0.100)	-40.24	-48.28	-52.00	-52.72	-51.60	-49.46	-46.79	-43.84
15	(0.025,0.200)	-38.34	-44.49	-46.26	-44.79	-41.16	-36.15	-30.25	-23.69
16	(0.050,0.200)	-74.71	-78.02	-78.96	-78.31	-76.62	-74.24	-71.39	-69.18
17	(0.100,0.100)	-96.39	-96.89	-97.04	-96.97	-96.76	-96.47	-96.14	-95.78
18	(0.125,0.125)	-98.56	-98.73	-98.77	-98.72	-98.60	-98.45	-98.29	-98.09

of acceptance in the presence of errors, i.e. $\frac{Pa_e - Pa}{Pa} \times 100$. The values given in Tables 20 and 21 are for a specific plan ($h_1 = 1.0458$, $h_2 = 1.3426$, $s = 0.0341$) found for (AQL = 0.01, $\alpha = 0.05$), (RQL = 0.08, $\beta = 0.10$) requirements. Figure 11 shows the OC curves in presence of inspection errors for few of the above error pairs and for the same plan ($h_1 = 1.0458$, $h_2 = 1.03426$, $s = 0.0341$).

The effect of inspection errors on the probability of acceptance is similar to that of single or double sampling plans. Probability of acceptance increases in presence of type II errors because the inspector is erroneously classifying defective items as good and hence accepting lots more often. Effect of type II errors is more pronounced at higher values of p, where the number of defectives that can be erroneously classified as good is more.

Type I error has an effect of decreasing the probability of acceptance at all incoming quality levels. The reason being the erroneous classification of good items as defectives. The effect of e_1 on Pa is more significant as compared to that of e_2. This is because we normally deal with good quality and the number of good items that can be classified as defectives is more. Percentage change is also more at higher value of p for e_1 and e_2 both. In case of e_2, it is because greater change (Pa_e - Pa) and in case of e_1, it is because of low values of Pa.

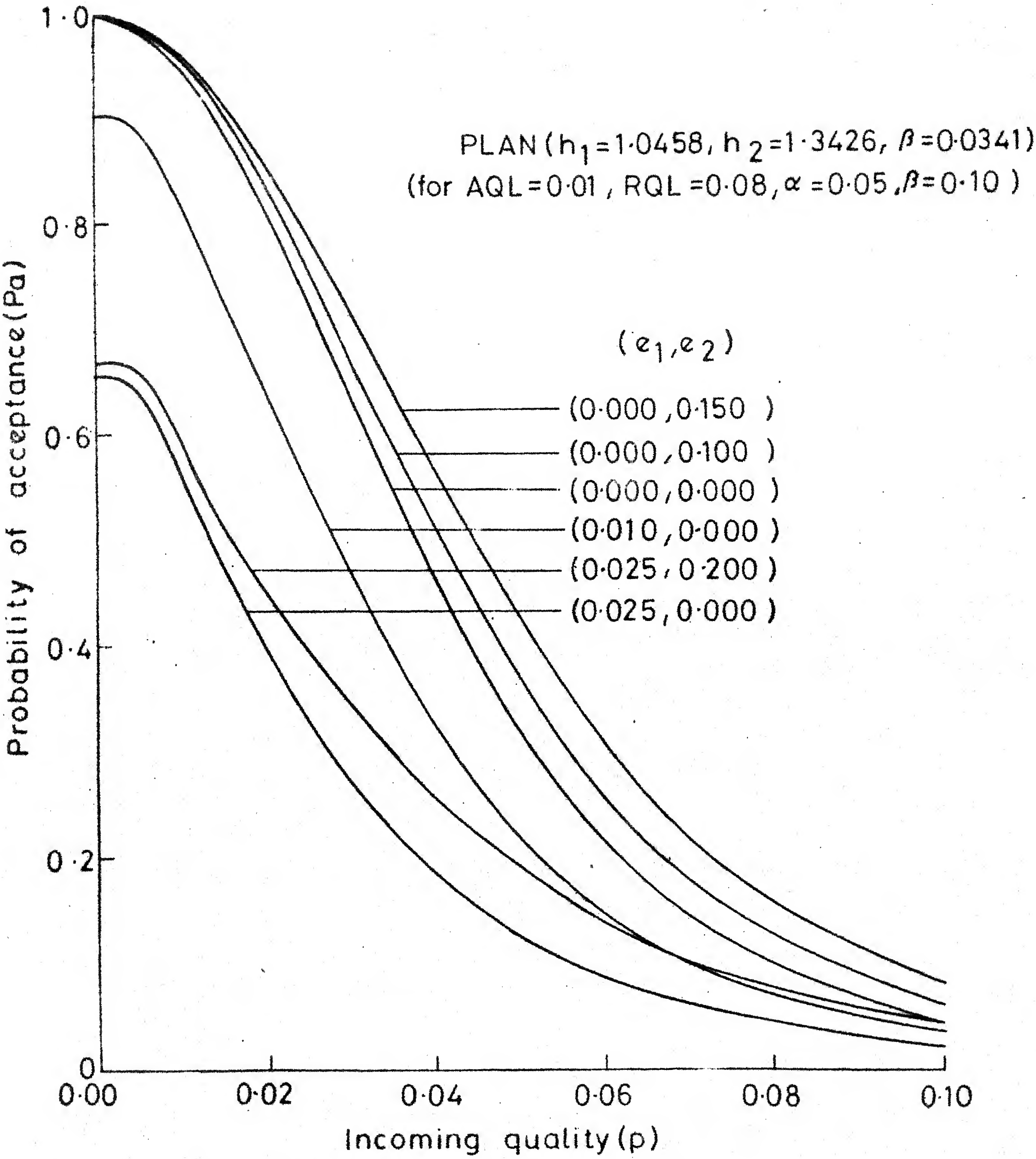

Fig.11 OC Curve in presence of inspection errors (Sequential sampling plan)

In presence of both type I and type II errors the net effect depends both upon relative values of e_1 and e_2 and the incoming quality. For low values of p, even if e_1 is much less than e_2, its effect is more dominating because the number of good items which will be subjected to type I errors is more than the number of defectives which will be subjected to type II errors.

4.2.3 Average Sample Number:

In sequential sampling, the inspector does not know exactly how many items he will have to examine from the lot. However, it is possible to compute the average amount of inspection that will be required in a long series of operations with a given sequential plan.

The average amount of inspection depends upon the values of h_1, h_2 and s which define the plan and also upon the quality of the lots submitted for inspection. The functional relationship between the average amount of inspection and these quantities is given below [The detailed analysis can be had from SRG report (13)].

Average sampling number for the lots which are accepted is given by,

$$ASN_a = \frac{h_1 + \left[\frac{p(1-p)}{Pa} \right] \frac{d\,Pa}{d\,p}}{s - p} \qquad (4.19)$$

Average sampling number for the lots which are rejected is given by,

$$ASN_r = \frac{h_2 + \left[\frac{p(1-p)}{1-Pa}\right]\frac{d\,Pa}{dp}}{p - s} \qquad (4.20)$$

The average sampling number (ASN) is given by,

$$ASN = ASN_a \cdot Pa + ASN_r\,(1 - Pa)$$

Substituting the values of ASN_a from Eq. (4.19) and ASN_r from Eq. (4.20), we get,

$$ASN = \frac{(h_1 + h_2)\,Pa - h_2}{s - p} \qquad (4.21)$$

In the presence of inspection errors Eq. (4.21) can be modified as follows,

$$ASN_e = \frac{(h_1 + h_2)\,Pa_e - h_2}{s - p_e} \qquad (4.22)$$

Table 22 gives the average sample number. Table 23 gives the percentage in average sample in the presence of errors, i.e. $\frac{ASN_e - ASN}{ASN} \times 100$. Fig. 12 shows the average sample number curve in absence or in presence of error for few of the above error pairs. The effects of inspection errors on the probability of acceptance and the probability of rejection, together determine the effect of errors on ASN. In presence of only type I error, $ASN_e > ASN$ for incoming quality better than some level p_{N_1} (say) beyond which

TABLE 22: AVERAGE SAMPLING NUMBER IN PRESENCE OF ERRORS(SEQUENTIAL SAMPLING PLAN)

PLAN: (h1=1.0458,h2=1.3426,s=0.0311)

(AQL=0.01, α=0.05,RQL=0.08, β=0.10)

	ERROR-PAIRS	INCOMING QUALITY 'p'							
	(e1,e2)	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
1	(0.000,0.000)	38.50	43.15	43.55	40.66	36.26	31.69	27.55	24.03
2	(0.000,0.050)	38.16	42.88	43.74	41.41	37.42	33.03	28.93	25.36
3	(0.000,0.075)	37.99	42.72	43.81	41.76	37.99	33.72	29.65	26.07
4	(0.000,0.100)	37.82	42.56	43.85	42.09	38.55	34.41	30.39	26.80
5	(0.000,0.125)	37.65	42.38	43.88	42.39	39.10	35.10	31.14	27.55
6	(0.000,0.150)	37.47	42.19	43.88	42.62	39.63	35.80	31.91	28.33
7	(0.000,0.200)	37.12	41.79	43.81	43.16	40.66	37.19	33.49	29.97
8	(0.010,0.000)	43.12	43.58	40.77	36.45	31.91	27.78	24.26	21.31
9	(0.025,0.000)	42.46	38.77	34.29	29.97	26.16	22.94	20.26	18.04
10	(0.050,0.000)	31.91	27.94	24.52	21.64	19.24	17.24	15.56	14.15
11	(0.075,0.000)	22.72	20.20	18.09	16.32	14.83	13.56	12.47	11.53
12	(0.100,0.000)	16.86	15.33	14.01	12.89	11.92	11.07	10.33	9.68
13	(0.010,0.100)	42.85	43.81	41.86	38.28	34.18	30.22	26.69	23.65
14	(0.050,0.100)	42.62	39.63	35.68	31.69	28.04	24.85	22.13	19.83
15	(0.025,0.200)	42.99	40.46	37.07	33.49	30.07	26.98	24.27	21.92
16	(0.050,0.200)	32.81	29.55	26.61	24.03	21.78	19.83	18.14	16.68
17	(0.100,0.100)	17.05	16.65	14.43	13.37	12.44	11.62	10.90	10.26
18	(0.125,0.125)	13.31	12.44	11.67	10.99	10.37	9.82	9.32	8.87

TABLE 23: PERCENTAGE CHANGE IN ASN FOR VARIOUS ERROR-PAIRS(SEQUENTIAL SAMPLING PLAN)

PLAN: ($h1=1.0458, h2=1.3426, s=0.0341$)

($AQL=0.01$, $\alpha=0.05$, $RQL=0.08$, $\beta=0.10$)

	ERROR-PAIRS	INCOMING QUALITY (p)							
	(e1,e2)	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	-0.88	-0.63	0.44	1.84	3.26	4.23	5.01	5.53
3	(0.000,0.075)	-1.32	-1.00	0.60	2.71	4.77	6.41	7.62	8.49
4	(0.000,0.100)	-1.77	-1.37	0.69	3.52	6.32	6.58	10.31	11.53
5	(0.000,0.125)	-2.21	-1.78	0.76	4.25	7.83	10.76	13.03	14.65
6	(0.000,0.150)	-2.68	-2.22	0.76	4.62	9.29	12.97	15.63	17.89
7	(0.000,0.200)	-3.58	-3.15	0.60	6.15	12.13	17.36	21.55	24.72
8	(0.010,0.000)	12.00	1.00	-6.38	-10.35	-12.00	-12.31	-11.94	-11.32
9	(0.025,0.000)	10.29	-10.15	-21.26	-26.29	-27.65	-27.61	-26.46	-24.93
10	(0.050,0.000)	-17.12	-35.25	-43.70	-46.78	-45.94	-45.60	-43.52	-41.12
11	(0.075,0.000)	-40.99	-53.19	-58.46	-59.86	-59.10	-57.21	-54.74	-52.02
12	(0.100,0.000)	-56.21	-64.17	-67.83	-68.30	-67.13	-65.07	-62.50	-59.72
13	(0.010,0.100)	11.30	1.53	-3.88	-5.85	-5.74	-4.64	-3.12	-1.58
14	(0.025,0.100)	10.70	-3.16	-18.07	-22.06	-22.57	-21.58	-19.67	-17.48
15	(0.025,0.200)	11.55	-6.23	-14.88	-17.63	-17.07	-14.66	-11.91	-8.78
16	(0.050,0.200)	-14.78	-31.52	-38.90	-40.90	-39.93	-37.43	-34.16	-30.59
17	(0.100,0.100)	-55.71	-63.73	-66.87	-67.17	-65.09	-63.33	-60.44	-57.30
18	(0.125,0.125)	-65.43	-71.17	-73.20	-72.97	-71.40	-69.01	-66.17	-63.09

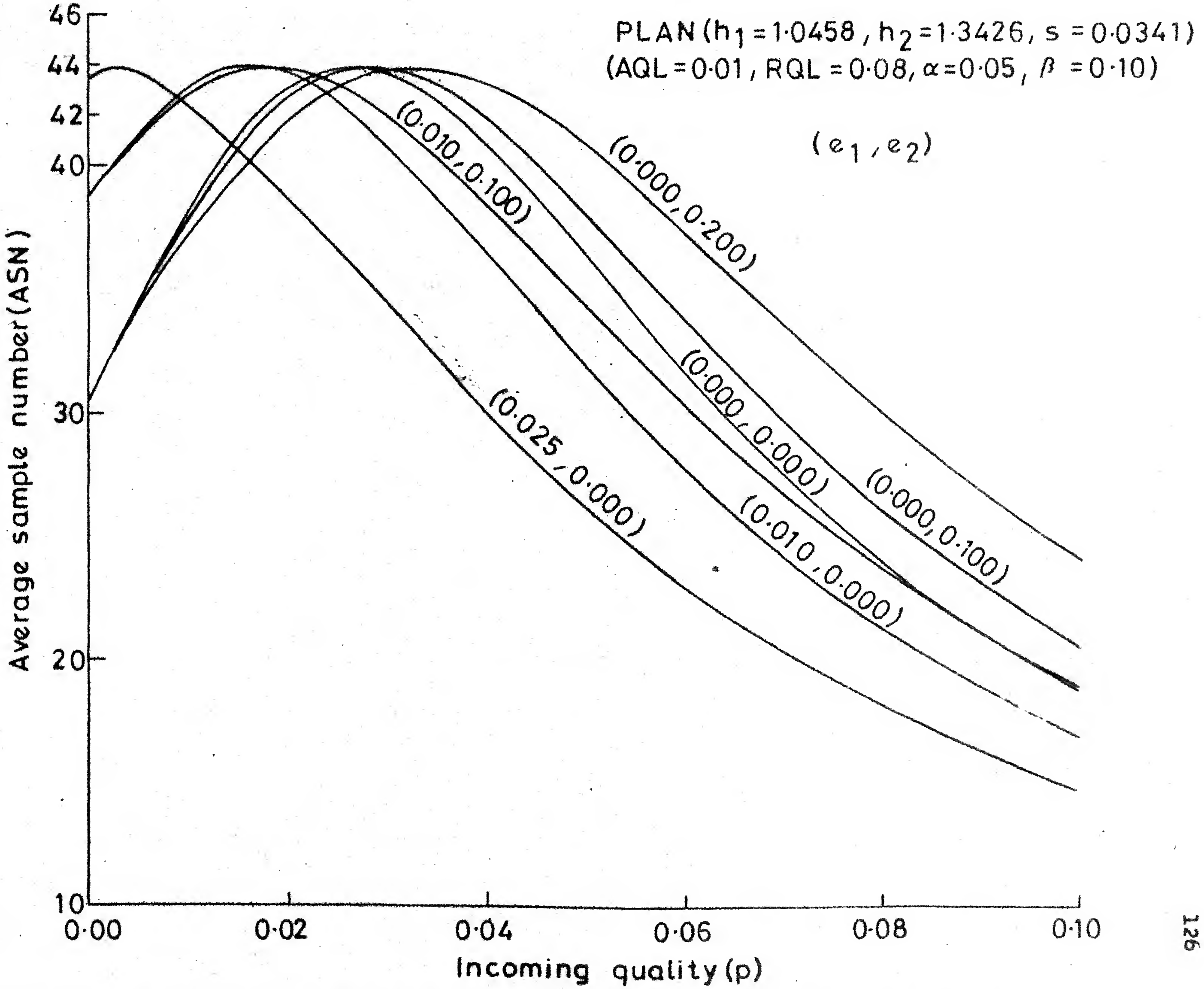

Fig.12 Average sample number in presence of inspection errors (Squential sampling plan)

$ASN_e < ASN$. Similarly in the presence of type two error, $ASN_e < ASN$ for incoming quality level p_{N_2} (Say) beyond which $ASN_e > ASN$.

p_{N_1} = Incoming quality at which type I error has no effect on ASN.

p_{N_2} = Incoming quality at which type II error has no effect on ASN.

p_N = Incoming quality at which there is no effect of inspection errors on ASN.

In presence of both type I and type II errors, there will be some quality level p_N at which the presence of error will have no effect on ASN. For quality levels better than p_N and worse than p_N the effect of errors depends upon relative values of type I and type II errors (i.e. e_1 and e_2). But for usual quality range e_1 is the dominating factor and hence the curve is expected to follow pattern of the curve in presence of type I error only i.e. for

$$p < p_N \quad ASN_e > ASN$$
$$p > p_N \quad ASN_e < ASN$$

To have a better understanding let us consider a double sampling plan, which had shown same effects of errors on ASN. ASN for a double sampling plan is given by Eq. (3.23) and Eq. (3.24).

$$ASN = n_1 + n_2 \, (1 - Pa_1 - Pr_1)$$
$$ASN_e = n_1 + n_2 \, (1 - Pa_{1e} - Pr_{1e})$$

When only type I error is present i.e. $e_1 > 0$, $e_2 = 0$, we have seen that $Pa_{e_1} < Pa_1$ and $Pre_1 > Pr_1$. For low values of p, Pa is a dominating member and hence,

$$Pa_{1e} + Pr_{1e} < Pa_1 + Pr_1 \quad , \quad p < p_{N_1}$$

$$Pa_{1e} + Pr_{1e} = Pa_1 + Pr_1 \quad , \quad \text{for } p = p_{N_1}$$

$$Pa_{1e} + Pr_{1e} > Pa_1 + Pr_1 \quad , \quad \text{for } p > p_{N_1}$$

these three relations when substituted in ASN expression given by Eq. (3.23),

$$ASN_e > ASN \qquad \text{for } p < p_{N_1}$$

$$ASN_e = ASN \qquad \text{for } p = p_{N_1}$$

$$ASN_e = ASN \qquad \text{for } p > p_{N_1}$$

Similarly, the relations can be developed for effect of errors in the presence of type II errors and written as,

$$ASN_e < ASN \qquad \text{for } p < p_{N_2}$$

$$ASN_e = ASN \qquad \text{for } p = p_{N_2}$$

$$ASN_e > ASN \qquad \text{for } p > p_{N_2}$$

The same reasoning holds good for sequential sampling plan also. The exact analysis for it is however, difficult.

4.2.4 Average Outgoing Quality:

When material is inspected by non destructive tests, lots which are rejected are often subjected to 100 percent inspection. This is known as rectification. Rectification of rejected lot may be done by replacing defectives found by good ones or by just scrapping the defectives. The average outgoing quality is then the quality of outgoing lots.

In case of sequential sampling plan, it is rather difficult and complex to determine the exact expression for the average outgoing quality even in absence of errors. However, the approximate expression is given by,

$$AOQ = Pa \cdot p \tag{4.24}$$

AOQ given by the approximate expression can be modified, replacing p by p_e and Pa by Pa_e, that is,

$$AOQ_e = Pa_e \; p_e \tag{4.25}$$

But the above equation is a bit too much of approximation hence further analysis of AOQ under inspection errors is not taken up.

4.2.5 Average Total Inspection:

In case of rectification, the rejected lots are subjected to 100 percent inspection. Therefore the average total amount of inspection (ATI) is much higher than average sample number discussed before.

It may be recalled that ASN was given in two parts. ASN_a for the accepted lots and ASN_r for the rejected lots. These expressions, given from Eq. (4.19) and (4.20) are as follows:

$$ASN_a = \frac{h_1 + \left[\frac{p\,(1 - p)}{Pa} \right] \frac{d\,Pa}{dp}}{s - p}$$

$$ASN_r = \frac{h_2 + \left[\frac{p\,(1 - p)}{(1 - Pa)} \right] \frac{d\,Pa}{dp}}{p - s}$$

In the case of rectification policy the average amount of inspection required for accepted lots is same as ASN_a. But when a lot is rejected all N items are inspected, and hence average amount of inspection is N. Then average total inspection (ATI) is given by,

$$ATI = ASN_a\,Pa + N\,(1 - Pa)$$

$$ATI = \frac{h_1 + \left[\frac{p(1 - p)}{Pa} \right] \frac{d\,Pa}{dp}}{s - p} \cdot Pa + N(1 - Pa) \qquad (4.26)$$

It may be noted that in the above equation, rectification with 100 percent inspection is considered and defectives found are either scrapped or replaced with good items from a prejudged lot without further inspection. In presence of inspection errors the Eq. (4.26) can be modified to the form given below.

$$ATI_e = \frac{h_1 + \left[\frac{p_e (1 - p_e)}{Pa_e} \right] \frac{d\, Pa_e}{d\, p_e}}{s - p_e} Pa_e + N (1 - Pa_e)$$

$$= \frac{h_1\, Pa_e + p_e (1 - p_e) \frac{d\, Pa_e}{d\, p_e}}{s - p_e} + N(1 - Pa\ ) \tag{4.27}$$

To evaluate the above expression, we should first have expression for $\frac{d\, Pa_e}{d\, p_e}$. This is given in Appendix A. For a given incoming quality p, we can get p_e, Pa_e and $\frac{d\, Pa_e}{dp}$. Thus ATI can be computed from Eq. (4.27).

The effects of inspection errors on ATI have to be studied. A typical sampling plan ($h_1 = 1.0458$, $h_2 = 1.3426$, $s = 0.0341$) is considered. ATI_e values are computed for various error pairs at selected incoming quality levels. The results are illustrated in Tables 24 and 25. Table 24 gives ATI_e values. Table 25 gives percentage change in average total inspection in presence of inspection errors i.e. $\frac{ATI_e - ATI}{ATI} \times 100$. Fig. 13 shows the ATI curves for few of the above error pairs. Again same plan ($h_1 = 1.0458$, $h_2 = 1.3426$, $s = 0.0341$) is considered.

The effect of e_1 and e_2 on average total inspection is same as in the case of single and double sampling plans. The values of ATI_e are for rectification policy without replacement. The effect of e_1 is to increase average total inspection because of more screening inspection and effect of e_2 is to decrease the average total inspection.

TABLE 24: AVERAGE TOTAL INSPECTION IN PRESENCE OF ERRORS(SEQUENTIAL SAMPLING PLAN)

PLAN: (h1=1.0456,h2=1.3426,s=0.0341,N=1000)

(AQL=0.01, α=0.05,RQL=0.08, β=0.10)

	ERROR-PAIRS (e1,e2)	INCOMING QUALITY 'p' 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
1	(0.000,0.000)	87.38	221.34	395.03	561.48	694.64	791.63	858.55	903.93
2	(0.000,0.050)	82.54	205.33	368.44	530.40	665.06	766.11	837.99	887.89
3	(0.000,0.075)	80.19	197.49	355.12	514.37	649.28	752.27	820.61	878.87
4	(0.000,0.100)	77.90	189.77	341.79	498.03	632.90	737.07	814.45	869.11
5	(0.000,0.125)	75.66	182.18	328.49	481.57	615.92	722.29	801.45	858.55
6	(0.000,0.150)	73.47	174.73	315.22	465.60	598.35	706.09	787.57	847.15
7	(0.000,0.200)	69.24	160.24	288.91	430.11	561.48	671.21	756.97	821.50
8	(0.010,0.000)	219.72	391.49	556.91	690.23	787.57	855.22	901.30	932.52
9	(0.025,0.000)	477.47	626.18	740.16	721.50	877.69	916.04	942.17	960.03
10	(0.050,0.000)	787.56	852.96	898.21	929.35	950.79	965.63	975.93	983.12
11	(0.075,0.000)	918.43	942.72	959.66	971.51	979.84	985.72	989.88	992.83
12	(0.100,0.000)	968.11	977.24	983.73	988.37	991.68	994.05	995.76	996.98
13	(0.010,0.100)	203.75	356.01	509.50	640.84	742.62	817.30	870.62	908.27
14	(0.050,0.100)	454.60	598.35	708.85	791.63	851.53	894.29	924.45	945.90
15	(0.025,0.200)	443.05	569.04	674.24	756.97	819.76	866.55	901.11	926.59
16	(0.050,0.200)	770.56	828.28	871.62	903.93	927.98	945.90	959.27	969.30
17	(0.100,0.100)	966.69	975.47	981.81	986.50	989.98	992.56	994.48	995.91
18	(0.125,0.125)	986.75	989.98	992.42	994.27	995.68	996.74	997.54	998.15

TABLE 25: PERCENTAGE CHANGE IN AIT FOR SELECTED ERROR-PAIRS (SEQUENTIAL SAMPLING PLAN)

PLAN: (h1=1.0458, h2=1.3426, s=0.0341, N=1000)

(AQL=0.01, α=0.05, RQL=0.08, β=0.10)

	ERROR-PAIRS (e1,e2)	INCOMING QUALITY (p) 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
1	(0.000,0.000)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	(0.000,0.050)	-5.54	-7.23	-5.73	-5.54	-4.29	-3.22	-2.39	-1.77
3	(0.000,0.075)	-8.22	-10.78	-10.10	-8.39	-6.56	-4.97	-3.72	-2.77
4	(0.000,0.100)	-10.85	-14.26	-13.48	-11.30	-8.91	-6.82	-5.14	-3.85
5	(0.000,0.125)	-13.41	-17.69	-16.84	-14.23	-11.36	-8.76	-6.65	-5.02
6	(0.000,0.150)	-15.92	-21.06	-20.20	-17.06	-13.89	-10.81	-8.27	-6.28
7	(0.000,0.200)	-20.76	-27.60	-26.86	-23.40	-19.19	-15.71	-11.83	-9.12
8	(0.010,0.000)	151.45	76.87	40.98	22.93	13.35	8.03	4.98	3.16
9	(0.025,0.000)	446.43	182.90	87.37	46.31	26.32	15.72	9.74	6.21
10	(0.050,0.000)	801.30	285.36	127.38	65.52	35.84	21.98	13.67	8.76
11	(0.075,0.000)	951.06	325.91	142.93	73.93	41.02	24.52	15.30	9.83
12	(0.100,0.000)	1007.93	341.51	149.03	76.93	42.72	25.57	15.98	10.29
13	(0.010,0.100)	133.18	60.84	28.98	14.13	6.88	3.24	1.41	0.48
14	(0.050,0.100)	420.49	170.33	79.44	40.99	22.55	12.96	7.68	4.64
15	(0.025,0.200)	407.04	157.09	70.68	34.82	17.98	9.46	4.96	2.51
16	(0.050,0.200)	781.85	274.21	120.65	60.99	33.55	19.49	11.73	7.23
17	(0.100,0.100)	1006.53	340.71	148.54	75.70	42.46	25.36	15.83	10.18
18	(0.125,0.125)	1029.26	347.27	151.23	77.06	43.30	25.91	16.19	10.42

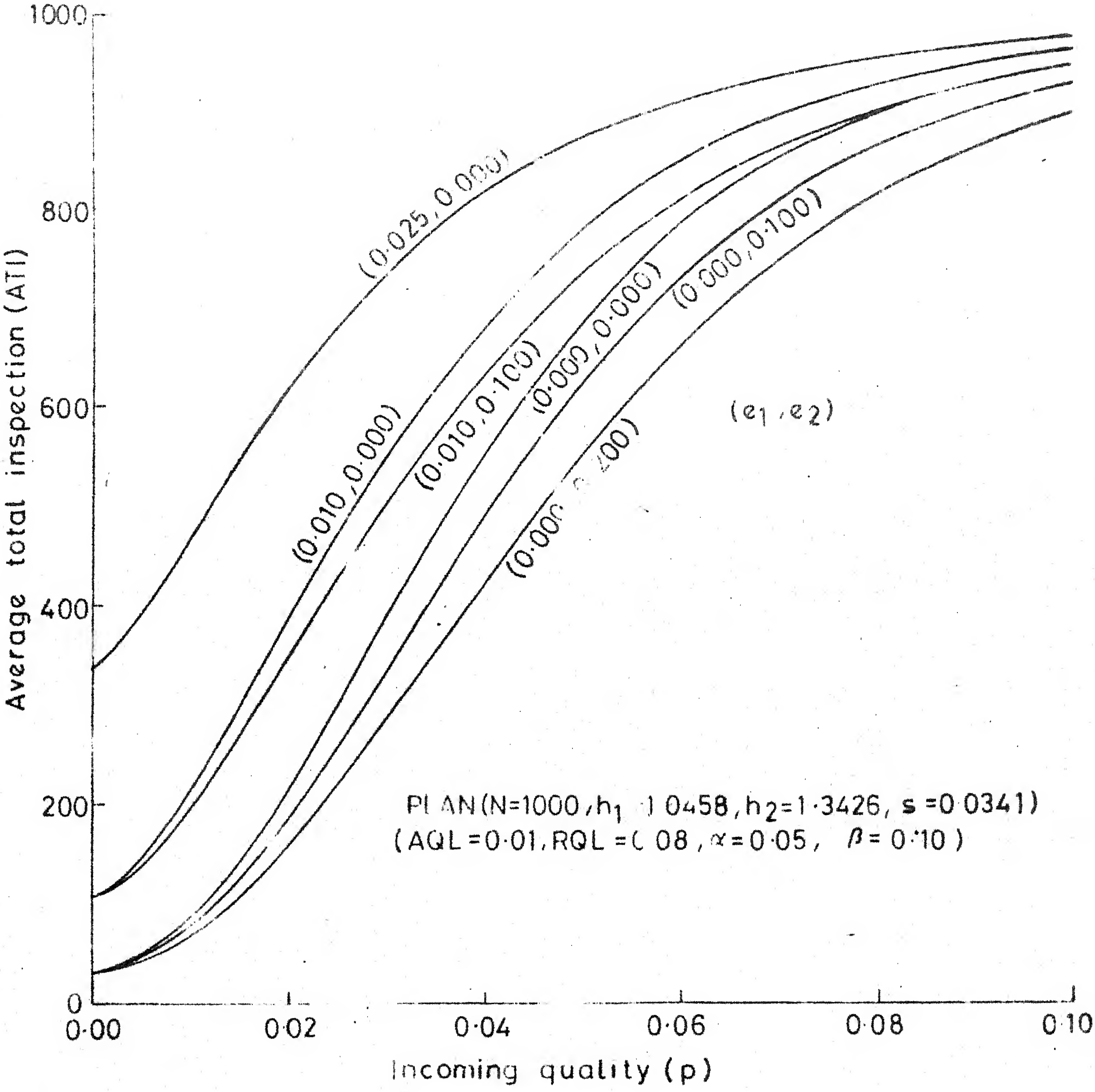

Fig. 13 Average total inspection in presence of inspection errors (Sequential sampling plan)

4.3 Design of Sequential Sampling Plan Based upon (AQL, α), (RQL, β) Requirements:

Let us first discuss the design procedure for case when inspection is error free.

When Inspection is Error Free:

We had obtained in previous section equations for acceptance and rejection lines defining a sequential sampling plan. Eq. (4.9) and (4.10) give acceptance line and rejection line respectively.

$$d_1 = -h_1 + s\,n \qquad \text{acceptance line}$$
$$d_2 = h_2 + s\,n \qquad \text{rejection line}$$

where h_1, h_2 and s are given by Eqs. (4.11), (4.12) and (4.13). Given (AQL, α), (RQL, β) we can find h_1, h_2 and s. Let AQL $= p_1$ and RQL $= p_2$ for simplicity. The three decision variables h_1, h_2 and s define the plan. The plan can be expressed in form of a table as shown in the Sec. (4.1) giving acceptance and rejection numbers. For each n, rounding off d_1 gives acceptance number and rounding up d_2 gives the rejection number.

When Inspection Error is Present:

When inspection error is present, the inspector is behaving as if incoming quality is p_e instead of p. With the assumption of Binomial distribution, we had seen in Sec. (2.2.1) that Binomial (n, p) gets modified to Binomial (n, p_e). Then

we can replace p by p_e in the likelihood ratio L given in Eq. (4.2) and the rest of the analysis can proceed on the same lines. We can therefore, get equivalent intercepts of decision lines h_{1e}, h_{2e} and slope s_e in presence of errors. The following relations determine the plan.

$$AQL_e = p_{1e} = p_1(1-e_2) + (1-p_1)\, e_1$$
$$RQL_e = p_{2e} = p_2(1-e_2) + (1-p_2)\, e_1 \qquad (4.28)$$

then,

$$h_{1e} = \frac{\log \frac{\beta}{(1-\alpha)}}{\log \left[\frac{(p_{2e}}{p_{1e}}\frac{(1-p_{1e})}{(1-p_{2e})}\right]}$$

$$h_{2e} = \frac{\log \left(\frac{1-\beta}{\alpha}\right)}{\log \left[\frac{p_{2e}}{p_{1e}}\frac{(1-p_{1e})}{(1-p_{2e})}\right]} \qquad (4.29)$$

$$s_e = \frac{\log \frac{1-p_{1e}}{1-p_{2e}}}{\log \left[\frac{p_{2e}}{p_{1e}}\frac{(1-p_{1e})}{(1-p_{2e})}\right]} \qquad (4.30)$$

The acceptance lines are given by,

$$d_{1e} = -h_{1e} + s_e\, n \qquad \text{acceptance line} \qquad (4.31)$$
$$d_{2e} = h_{2e} + s_e\, n \qquad \text{rejection line} \qquad (4.32)$$

The decision to accept the lot is taken if the number of observed defectives is less than or equal to d_{1e} given by Eq. (2.31). The decision to reject the lot is taken if the

number of observed defectives is more than or equal to d_{2e} given by Eq. (2.32).

It may be noted that the above designed compensating plan which also accounts for errors, will have the same specified risks at AQL and RQL. That means the observed OC curve with the erroneous inspector will fit the desired OC curve at p_1 and p_2. The rest of the curve be different than the one desired.

Effect of Errors on h_1, h_2 and s:

Let us try to see analytically what the effect of errors on the above parameters would be.

Case 1: When only type I error is present, i.e.

$$e_1 > 0$$
$$e_2 = 0$$

Given, p_1, p_2, α, β, e_1, e_2

$$p_{1e} = p_1(1-e_2) + (1-p_1)\, e_1$$

$$p_{1e} = p_1(1-e_1) + e_1 \quad (\text{as } e_2 = 0)$$

Similarly, (4.33)

$$p_{2e} = p_2(1-e_1) + e_1$$

From Eqns. (4.28), (4.11) we get,

$$h_{1e} = \frac{\log \frac{\beta}{1-\alpha}}{\log\left[\frac{p_{2e}\,(1-p_{1e})}{p_{1e}\,(1-p_{2e})}\right]}$$

$$h_1 = \frac{\log \frac{\beta}{1-\alpha}}{\log \left[\frac{p_2 \; (1-p_1)}{p_1 \; (1-p_2)}\right]}$$

$$1-p_{1e} = 1 - p(1 - e_1) - e_1 .$$

$$= (1 - p_1)(1 - e_1)$$

$$1 - p_{2e} = (1 - p_2) \; (1 - e_1)$$

$$\frac{1 - p_{1e}}{1 - p_{2e}} = \frac{1 - p_1}{1 - p_2} \tag{4.34}$$

$$\frac{p_{2e}}{p_{1e}} = \frac{p_2 \; (1-e_1) + e_1}{p_1 \; (1-e_1) + e_1}$$

$$\frac{p_{2e}}{p_{1e}} < \frac{p_2}{p_1} \quad \text{is true.}$$

If $$\frac{p_2(1-e_1) + e_1}{p_1(1-e_1) + e_1} < \frac{p_2}{p_1}$$

$$p_2p_1 \; (1-e_1) + p_1e_1 < p_2p_1 \; (1-e_1) + p_2e_1$$

$$p_1e_1 < p_2e_1$$

$$p_1 < p_2$$

which is true hence,

$$\frac{p_{2e}}{p_{1e}} < \frac{p_2}{p_1} \tag{4.35}$$

Multiplying Eq. (4.35) on both sides by Eq. (4.34), we get,

$$\frac{p_{2e} \; (1 - p_{1e})}{p_{1e} \; (1 - p_{2e})} < \frac{p_2 \; (1 - p_1)}{p_1 \; (1 - p_2)}$$

taking logarithms we get,

$$\log \left[\frac{p_{2e}\ (1-p_{1e})}{p_{1e}\ (1-p_{2e})} \right] < \log \left[\frac{p_2\ (1-p_1)}{p_1\ (1-p_2)} \right] \tag{4.36}$$

then we get $h_{1e} > h_1$, $h_{2e} > h_{1e}$ and $s_e > s$, because Eq.(4.36) gives relationship between the denominators of h_{1e} and h_1, h_{2e} and h_2, s and s_e.

Case II: When only type II error is present, i.e.

$$e_1 = 0, \quad e_2 > 0$$

$$p_{1e} = p_1\ (1 - e_2)$$

$$p_{2e} = p_2\ (1 - e_2)$$

$$\frac{p_{2e}}{p_{1e}} = \frac{p_2}{p_1} \tag{4.37}$$

$$\frac{1 - p_{1e}}{1 - p_{2e}} < \frac{1 - p_1}{1 - p_2} \quad \text{is true}$$

if $$\frac{1 - p_1\ (1 - e_2)}{1 - p_2\ (1 - e_2)} < \frac{1 - p_1}{1 - p_2}$$

$$1 - p_2 - p_1\ (1 - e_2) + p_2 p_1\ (1 - e_2)$$

$$< 1 - p_1 + p_1 p_2\ (1-e_2) - p_2(1-e_2)$$

$$-p_2 - p_1 + p_1 e_2 < -p_1 - p_2 + p_2 e_2$$

$$p_1 < p_2 \ , \quad \text{which is true.}$$

Hence,

$$\frac{1 - p_{1e}}{1 - p_{2e}} < \frac{1 - p_1}{1 - p_2} \tag{4.38}$$

Eq. (4.38) gives relative values of denominator of h_{1e} and h_1, h_{2e} and h_2. So we get,

$$h_{1e} > h_1 \quad \text{and} \quad h_{2e} > h_2 \tag{4.39}$$

$$\frac{p_{2e}}{p_{1e}} \frac{1-p_{1e}}{1-p_{2e}} < \frac{p_2}{p_1} \frac{1-p_1}{1-p_2} \quad \text{(From Eqs. (4.37), (4.38))}$$

Let,

$$A = \log \frac{1 - p_{1e}}{1 - p_{2e}}, \qquad B = \log \frac{p_{2e}}{p_{1e}}$$

$$A' = \log \frac{1 - p_1}{1 - p_2}, \qquad B' = \log \frac{p_2}{p_1}$$

$B' = B$, and $A < A'$ from Eqs. (4.37) and (4.38);

$s_e < s$ is true.

If,

$$\frac{\log \dfrac{1 - p_{1e}}{1 - p_{2e}}}{\log \left[\dfrac{p_{2e}}{p_{1e}} \dfrac{(1 - p_{1e})}{(1 - p_{2e})}\right]} < \frac{\log \dfrac{1 - p_1}{1 - p_2}}{\log \left[\dfrac{p_2}{p_1} \dfrac{(1 - p_1)}{(1 - p_2)}\right]}$$

$$\frac{A}{B + A} < \frac{A'}{B' + A'}$$

$$AB' + AA' < A'B' + AA'$$

$$AB' < A'B'$$

$$A < A'$$

This is true as given above, and hence

$$s_e < s \tag{4.40}$$

To study the effect of inspection errors on the plan, various error pairs were selected along with a few sets of [(AQL, α), (RQL, β)] requirements. Plans are obtained in terms of h_{1e}, h_{2e}, s_e. The results are given in Table 26.

The effect of e_1 and e_2 sequential sampling plan can be studied from Table 26. In presence of e_1, h_1, h_2 and s all three increase. In presence of e_1, h_1 and h_2 increase but slope s decreases. These results have been explained analytically in the preceeding analysis. The effect if expressed in terms of acceptance and rejection numbers is similar to that in case of single and double sampling plans. In presence of e_1 the acceptance and rejection numbers are higher for a few sample numbers to compensate for good items erroneously classified. In the presence of e_2 same acceptance and rejection numbers are valid for the higher values of n because some defectives might have been erroneously classified as good. The words 'Few values of n' indicate that acceptance and rejection numbers do not change for all values of n. It is so because of the integer requirements. The average effect of e_1 or e_2 on inspection of single item is too low and is absorbed in rounding up or rounding off changes in acceptance and rejection numbers.

4.4 Comparison of Single, Double and Sequential Sampling Plans in Presence of Errors:

In the previous two chapters and in the present chapter, it has been seen that inspection errors do affect the performance

[illegible] SEQUENTIAL SAMPLING PLANS IN PRESENCE OF ERRORS AND VARIOUS VALUES OF AQL,RQL
$\alpha=0.05,\beta=0.10$

[illegible] (e1,e2)	AQL/RQL [illegible]	0.0170.10	0.0170.12	0.0170.14	0.0170.16	0.0270.16	0.0370.16	0.0470.16
([illegible])	n1=1.[illegible] n2=1.[illegible]26 s=[illegible]	n1=1.9339 n2=1.2253 s=0.0397	n1=0.8650 n2=1.11[illegible]5 s=0.0453	n1=0.8099 n2=1.0398 s=0.0506	n1=0.7665 n2=0.9842 s=0.0559	n1=1.0079 n2=1.2940 s=0.0690	n1=1.2384 n2=1.5900 s=0.0792	n1=1.4813 n2=1.9018 s=0.0879
([illegible])	n1=1.[illegible] n2=1.3450 s=0.0323	n1=0.9418 n2=1.2274 s=0.0377	n1=0.8671 n2=1.1132 s=0.0429	n1=0.8121 n2=1.0398 s=0.0480	n1=0.7689 n2=0.9872 s=0.0531	n1=1.0118 n2=1.2990 s=0.0655	n1=1.2439 n2=1.5969 s=0.0751	n1=1.4885 n2=1.9111 s=0.0834
([illegible])	n1=1.[illegible] n2=1.3452 s=0.0315	n1=0.941[illegible] n2=1.2192 s=0.0367	n1=0.8681 n2=1.1146 s=0.0418	n1=0.8132 n2=1.0440 s=0.0467	n1=0.7701 n2=0.9887 s=0.0516	n1=1.0137 n2=1.3014 s=0.0637	n1=1.2466 n2=1.6004 s=0.0731	n1=1.4921 n2=1.9157 s=0.0812
([illegible])	n1=1.0495 n2=1.3471 s=0.03[illegible]	n1=0.9428 n2=1.2104 s=0.0357	n1=0.8692 n2=1.1159 s=0.0406	n1=0.8143 n2=1.0455 s=0.0454	n1=0.7712 n2=0.9902 s=0.0502	n1=1.0156 n2=1.3039 s=0.0619	n1=1.2493 n2=1.6039 s=0.0711	n1=1.4958 n2=1.9204 s=0.0789
([illegible],0.125)	n1=1.[illegible]504 n2=1.3486 s=0.02[illegible]	n1=0.9438 n2=1.2117 s=0.0347	n1=0.8702 n2=1.1172 s=0.0395	n1=0.8154 n2=1.0469 s=0.0441	n1=0.7724 n2=0.9917 s=0.0487	n1=1.0175 n2=1.3063 s=0.0602	n1=1.2520 n2=1.6074 s=0.0691	n1=1.4994 n2=1.9250 s=0.0767
([illegible],0.150)	n1=1.[illegible]514 n2=1.3498 s=0.02[illegible]	n1=0.9446 n2=1.2130 s=0.0337	n1=0.8713 n2=1.1186 s=0.0393	n1=0.8165 n2=1.0483 s=0.0429	n1=0.7736 n2=0.9932 s=0.0473	n1=1.0194 n2=1.3088 s=0.0584	n1=1.2547 n2=1.6108 s=0.0671	n1=1.5030 n2=1.9296 s=0.0745
(0.000,0.200)	n1=1.0532 n2=1.3522 s=0.0272	n1=0.9467 n2=1.2155 s=0.0317	n1=0.8733 n2=1.1213 s=0.0360	n1=0.8187 n2=1.0511 s=0.0403	n1=0.7759 n2=0.9962 s=0.0444	n1=1.0232 n2=1.3136 s=0.0549	n1=1.2601 n2=1.6178 s=0.0631	n1=1.5102 n2=1.9389 s=0.0701
(0.010,0.000)	n1=1.4306 n2=1.8369 s=0.0456	n1=1.2535 n2=1.6094 s=0.0531	n1=1.1340 n2=1.4559 s=0.0593	n1=1.0465 n2=1.3435 s=0.0654	n1=0.9789 n2=1.2567 s=0.0714	n1=1.1937 n2=1.5326 s=0.0817	n1=1.4169 n2=1.8191 s=0.0906	n1=1.6604 n2=2.1317 s=0.0985
(0.025,0.000)	n1=1.9410 n2=2.4920 s=0.0632	n1=1.6312 n2=2.1327 s=0.0703	n1=1.4758 n2=1.8948 s=0.0772	n1=1.3424 n2=1.7234 s=0.0839	n1=1.2406 n2=1.5928 s=0.0905	n1=1.4458 n2=1.8562 s=0.0990	n1=1.6691 n2=2.1429 s=0.1067	n1=1.9190 n2=2.4638 s=0.1138
(0.050,0.000)	n1=2.7339 n2=3.5093 s=0.0890	n1=2.2831 n2=2.9312 s=0.0967	n1=1.9893 n2=2.5541 s=0.1041	n1=1.7807 n2=2.2862 s=0.1113	n1=1.6236 n2=2.0845 s=0.1185	n1=1.8328 n2=2.3531 s=0.1255	n1=2.0672 n2=2.6540 s=0.1321	n1=2.3342 n2=2.9969 s=0.1385
(0.075,0.000)	n1=3.4986 n2=4.4917 s=0.1140	n1=2.8770 n2=3.6937 s=0.1213	n1=2.4747 n2=3.1771 s=0.1295	n1=2.1909 n2=2.8129 s=0.1370	n1=1.9788 n2=2.5405 s=0.1444	n1=2.1999 n2=2.8244 s=0.1506	n1=2.4503 n2=3.1459 s=0.1566	n1=2.7381 n2=3.5154 s=0.1624
(0.100,0.000)	n1=4.2519 n2=5.4589 s=0.1385	n1=3.4583 n2=4.4400 s=0.1464	n1=2.9468 n2=3.7833 s=0.1542	n1=2.5878 n2=3.3224 s=0.1618	n1=2.3206 n2=2.9794 s=0.1694	n1=2.5564 n2=3.2821 s=0.1750	n1=2.8251 n2=3.6271 s=0.1806	n1=3.1353 n2=4.0253 s=0.1860
(0.010,0.100)	n1=1.4776 n2=1.8974 s=0.0431	n1=1.2930 n2=1.6601 s=0.0489	n1=1.1686 n2=1.5004 s=0.0546	n1=1.0779 n2=1.3839 s=0.0601	n1=1.0079 n2=1.2940 s=0.0555	n1=1.2244 n2=1.5720 s=0.0746	n1=1.4509 n2=1.8627 s=0.0825	n1=1.6987 n2=2.1810 s=0.0895
(0.025,0.100)	n1=2.0459 n2=2.6267 s=0.0595	n1=1.7466 n2=2.2424 s=0.0660	n1=1.5489 n2=1.9886 s=0.0722	n1=1.4070 n2=1.8064 s=0.0783	n1=1.2991 n2=1.6679 s=0.0842	n1=1.5076 n2=1.9355 s=0.0917	n1=1.7356 n2=2.2286 s=0.0985	n1=1.9922 n2=2.5577 s=0.1048
(0.025,0.200)	n1=2.1757 n2=2.7934 s=0.0558	n1=1.8517 n2=2.3773 s=0.0616	n1=1.6384 n2=2.1035 s=0.0671	n1=1.4858 n2=1.9076 s=0.0726	n1=1.3702 n2=1.7591 s=0.0779	n1=1.5825 n2=2.0317 s=0.0844	n1=1.8164 n2=2.3320 s=0.0903	n1=2.0801 n2=2.6706 s=0.0958
(0.050,0.200)	n1=3.1886 n2=4.0940 s=0.0812	n1=2.6460 n2=3.3971 s=0.0873	n1=2.2938 n2=2.9450 s=0.0933	n1=2.0451 n2=2.6256 s=0.0992	n1=1.8588 n2=2.3865 s=0.1050	n1=2.0835 n2=2.6750 s=0.1102	n1=2.3371 n2=3.0006 s=0.1153	n1=2.6278 n2=3.3738 s=0.1202
(0.100,0.100)	n1=4.6652 n2=5.9895 s=0.1344	n1=3.7838 n2=4.8579 s=0.1415	n1=3.2168 n2=4.1300 s=0.1484	n1=2.8197 n2=3.6201 s=0.1553	n1=2.5247 n2=3.2414 s=0.1620	n1=2.7752 n2=3.5630 s=0.1670	n1=3.0611 n2=3.9301 s=0.1718	n1=3.3918 n2=4.3546 s=0.1766
(0.125,0.125)	n1=5.6822 n2=7.2952 s=0.1576	n1=4.5689 n2=5.8659 s=0.1644	n1=3.8550 n2=4.9494 s=0.1711	n1=3.3565 n2=4.3093 s=0.1778	n1=2.9875 n2=3.8356 s=0.1843	n1=3.2635 n2=4.1900 s=0.1888	n1=3.5798 n2=4.5960 s=0.1931	n1=3.9465 n2=5.0668 s=0.1974

SEQUENTIAL SAMPLING PLAN (h1,h2,s)

measures significantly. At this stage we study the sensitivity of the three plans to the two types of errors. For this purpose the three plans are compared as regarding the OC curve or the probability of acceptance. First of all the three plans are designed for some (AQL, α), (RQL, β) requirements in absence of errors. It may, however, be noted that it is rather impossible to design the plans satisfying the requirements exactly, three plans may not have exactly the same OC curve even in the absence of inspection errors. Hence the three plans are designed to satisfy the above requirements to the best possible extent. Then the effects of errors on probability of acceptance for these plans is studied. The plans are designed for (AQL = 0.02, α = 0.05), (RQL = 0.15, β=0.10) requirements. The results are given in Tables 27 and 28. Table 27 gives the probability of acceptance at various incoming quality levels. Few representative error pairs are considered. Table 28 gives the percentage change in probability of acceptance in presence of errors, i.e. $(Pa_e - Pa)/Pa \times 100$. The same is repeated for each of the three plans.

The results indicate that in presence of type II error only for a good incoming quality ($p < 0.1$), single sampling plan is least sensitive to errors followed by double sampling plan and sequential sampling plan, in that order. But in case of $p > 0.1$ sequential plan is the best, followed by single and double sampling plan, in that order.

Table 27: Probability of Acceptance for Single, Double and Sequential Sampling Plans in Presence of Errors.

(e_1, e_2)	0.01	0.03	0.05	0.07	0.09	0.11	0.13	0.15	0.17	
0.00, 0.00	0.9953	0.9188	0.7593	0.5714	0.3989	0.2620	0.1634	0.0975	0.0558	S1
	0.9867	0.8925	0.7328	0.5500	0.3820	0.2486	0.1533	0.0903	0.0512	S2
	0.9867	0.8901	0.7177	0.5243	0.3587	0.2370	0.1543	0.1000	0.0648	S3
0.025, 0.00	0.8866	0.7199	0.5369	0.3740	0.2462	0.1545	0.0930	0.0539	0.0301	S1
	0.8591	0.6942	0.5165	0.3577	0.2333	0.1447	0.0860	0.0494	0.0275	S2
	0.8543	0.6763	0.4902	0.3359	0.2235	0.1470	0.0963	0.0631	0.0413	S3
0.05, 0.00	0.6703	0.4945	0.3429	0.2259	0.1424	0.0863	0.0505	0.0285	0.0156	S1
	0.6462	0.4752	0.3274	0.2136	0.1331	0.0797	0.0462	0.0260	0.0143	S2
	0.6248	0.4489	0.3079	0.2063	0.1370	0.0907	0.0601	0.0398	0.0263	S3
0.10, 0.00	0.2679	0.1759	0.1114	0.0682	0.0404	0.0233	0.0131	0.0071	0.0038	S1
	0.2543	0.1653	0.1035	0.0627	0.0369	0.0212	0.0120	0.0066	0.0036	S2
	0.2421	0.1646	0.1115	0.0754	0.0510	0.0345	0.0233	0.0158	0.0106	S3
0.00, 0.05	0.9959	0.9280	0.7820	0.6041	0.4350	0.2957	0.1915	0.1189	0.0711	S1
	0.9879	0.9023	0.7550	0.5818	0.4171	0.2814	0.1803	0.1106	0.0654	S2
	0.9879	0.9004	0.7416	0.5571	0.3922	0.2662	0.1775	0.1177	0.0779	S3
0.00, 0.10	0.9965	0.9367	0.8041	0.6371	0.4728	0.3324	0.2233	0.1442	0.0899	S1
	0.9891	0.9116	0.7767	0.6139	0.4540	0.3171	0.2111	0.1348	0.0831	S2
	0.9891	0.9104	0.7652	0.5907	0.4281	0.2985	0.2041	0.1385	0.0937	S3
0.00, 0.15	0.9970	0.9448	0.8256	0.6703	0.5122	0.3721	0.2591	0.1738	0.1128	S1
	0.9903	0.9206	0.7979	0.6462	0.4924	0.3558	0.2457	0.1632	0.1049	S2
	0.9902	0.9198	0.7881	0.6248	0.4660	0.3342	0.2345	0.1629	0.1127	S3

contd... 1

Table 27 continued

(e_1, e_2)	0.01	0.03	0.05	0.07	0.09	0.11	0.13	0.15	0.17	
0.025, 0.100	0.8938	0.7479	0.5830	0.4288	0.3005	0.2021	0.1310	0.0822	0.0501	S1
	0.8664	0.7216	0.5613	0.4112	0.2861	0.1905	0.1222	0.759	0.0459	S2
	0.8622	0.7055	0.5360	0.3865	0.2704	0.1863	0.1277	0.0873	0.0597	S3
0.050, 0.200	0.6893	0.5483	0.4187	0.3087	0.2207	0.1536	0.1042	0.0691	0.0449	S1
	0.6646	0.5276	0.4013	0.2940	0.2086	0.1438	0.0967	0.0636	0.0410	S2
	0.6444	0.5015	0.3770	0.2775	0.2020	0.1462	0.1056	0.0762	0.0551	S3
0.100, 0.100	0.2739	0.1892	0.1269	0.0828	0.0527	0.0328	0.0199	0.0118	0.0069	S1
	0.1438	0.0967	0.0636	0.0410	0.0260	0.0162	0.0100	0.0061	0.0047	S2
	0.2472	0.1756	0.1242	0.0878	0.0620	0.0438	0.0310	0.0219	0.0154	S3

NOTE: S1 - Single Sampling Plan
S2 - Double Sampling Plan
S3 - Sequential Sampling Plan

Table 28: Percentage change in Probability of Acceptance for Single, Double and Sequential Sampling Plans for Various Error Pairs.

(e_1, e_2)	Incoming Quality (p)									
	0.01	0.03	0.05	0.07	0.09	0.11	0.13	0.15	0.17	
0.00, 0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	S1
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	S2
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	S3
0.025, 0.00	-10.92	-21.65	-29.29	-34.55	-38.28	-41.03	-43.08	-44.72	-46.06	S1
	-12.93	-22.22	-29.52	-34.96	-38.93	-41.79	-43.90	-45.29	-46.29	S2
	-13.42	-24.02	-31.70	-35.94	-37.69	-37.97	-37.59	-37.00	-36.27	S3
0.050, 0.00	-32.65	-46.18	-54.84	-60.47	-64.30	-67.06	-69.09	-70.77	-72.04	S1
	-34.51	-46.76	-55.32	-61.16	-65.16	-67.94	-69.86	-71.21	-72.07	S2
	-36.68	-49.57	-57.10	-60.65	-61.81	-61.73	-61.05	-60.23	-59.41	S3
0.100, 0.00	-73.08	-80.86	-85.33	-88.06	-89.87	-91.10	-91.98	-92.72	-93.19	S1
	-74.23	-81.48	-85.88	-88.60	-90.34	-91.47	-92.17	-92.69	-92.97	S2
	-75.46	-81.51	-84.47	-85.62	-85.77	-85.44	-84.90	-84.25	-83.64	S3
0.00, 0.05	0.06	1.00	2.99	5.72	9.05	12.86	17.20	21.94	27.42	S1
	0.12	1.09	3.03	5.78	9.19	13.19	17.61	22.48	27.73	S2
	0.12	1.16	3.33	6.26	9.34	12.32	15.04	17.68	20.21	S3
0.00, 0.10	0.12	1.95	5.90	11.50	18.53	26.87	36.66	47.90	61.11	S1
	0.24	2.14	5.99	11.62	18.85	27.55	37.70	49.28	62.30	S2
	0.24	2.28	6.62	12.66	19.35	25.95	32.27	38.46	44.60	S3
0.00, 0.15	0.17	2.83	8.73	17.31	28.40	42.02	58.57	78.26	102.15	S1
	0.36	3.15	8.88	17.49	28.90	43.14	60.27	80.73	104.88	S2
	0.35	3.33	9.81	19.17	29.91	41.01	51.98	62.85	73.92	S3

contd...

Table 28 contd...

(e_1, e_2)	Incoming Quality (p)									
	0.01	0.03	0.05	0.07	0.09	0.11	0.13	0.15	0.17	
0.025, 0.100	-10.20	-18.60	-33.22	-24.96	-24.67	-22.86	-19.83	-15.69	-10.22	S1
	-12.19	-19.15	-23.40	-25.24	-25.10	-23.37	-20.29	-15.95	-10.35	S2
	-12.63	-20.74	-25.32	-26.84	-24.62	-21.39	-19.24	-12.69	- 7.87	S3
0.050, 0.200	-30.74	-40.32	-44.86	-45.97	-44.67	-41.37	-36.23	-29.13	-19.53	S1
	-32.64	-40.90	-45.24	-46.55	-45.39	-42.16	-36.92	-29.57	-19.92	S2
	-34.69	-43.66	-47.47	-47.07	-43.69	-38.31	-31.56	-24.76	-14.97	S3
0.100, 0.100	-72.48	-79.41	-83.29	-85.51	-86.71	-87.48	-87.82	-87.90	-87.63	S1
	-85.43	-89.17	-91.32	-92.55	-93.19	-93.48	-93.48	-93.24	-90.82	S2
	-74.95	-80.27	-82.69	-83.25	-82.72	-86.16	-79.91	-79.13	-76.23	S3

NOTE: S1 - Single Sampling Plan
S2 - Double Sampling Plan
S3 - Sequential Sampling Plan

In case only type I is present for a very good incoming quality ($p < 0.03 - 0.05$), again single sampling plan is least sensitive followed by double and sequential sampling plans in that order. But for $p > 0.3 - 0.5$ sequential sampling plan becomes least sensitive followed by single and double sampling plans.

When both the errors are present the order remains the same as in presence of type I error only, because type I error is the dominating member amongst the two.

In short, double sampling plan is the most sensitive one for all but a few incoming quality levels. Sequential sampling plan is the least sensitive except when incoming quality is good ($p < 0.1$). Single sampling is best for $p < 0.25$ while for the rest of the range of incoming quality, single sampling plan is more sensitive than the sequential sampling plan but less sensitive than the double sampling plan to the presence of inspection errors.

The observations made above are for a typical set of requirements, and may not be generalized for the sensitivity of OC curves of the three plans to the presence of errors. The methodology presented of course, can be used to study the sensitivity for any given set of requirements.

4.5 Conclusions:

Following conclusions can be drawn from this chapter.

1. Probability of acceptance increases in presence of type II error and decreases in presence of type I error. This is similar to the case of other two plans.

2. Expression for average outgoing quality is rather complex and hence approximate expression (given by AOQ = Pa.p) is generally used. This expression should not be modified, just by replacing Pa_e for Pa and p_e for p or else it will be too much of approximation.

3. ASN and ATI in the presence of error show similar pattern as in case of double sampling plan.

4. A design procedure has been suggested to compensate for the presence of errors, based on (AQL, α), (RQL, β) requirements. The intercepts h_1 and h_2 increase in presence of both, type I error and type II error. Slope s decreases in presence of type I error. This effect when transferred to acceptance and rejection number follows a pattern similar to other plans.

5. How single, double and sequential sampling plans compare in presence of errors is discussed. The sensitivity of the three plans on the basis of OC curve , to the presence of inspection errors is studied. The results are for a typical set of requirements,hence not generalized, however, the methodology presented can be used to study the sensitivity for any given set of requirements.

CHAPTER V

MEASUREMENT ERRORS AND VARIABLES ACCEPTANCE SAMPLING

5.1 Introduction:

5.1.1 Variables Acceptance Sampling:

Acceptance sampling by variables is one of the basic quality control techniques, used to determine the acceptability of the product, when a quality characteristic is measurable on a continuous scale and is known to have a distribution of a specific type, say normal. It may be possible to use a sampling plan based on the sample measurement such as the mean of the sample or the mean and standard deviation of the sample.

5.1.2 Measurement Errors:

One of the assumption implicit in the development of model for variables sampling plans is that the dimensions observed are true representation of the actual dimension. Unfortunately no method of measurement is absolutely accurate or fool proof. Although steps can be taken to increase the inspection accuracy, the fact remains that measurement errors will not be eliminated. The two error factors of primary concern in variables measurement are Bias and Imprecision. Each of the two has its own importance and both together characterize the measurement errors.

Bias:

Bias is the difference between true dimension of a product and the average of a long series of repeated measurements made on that product. Bias will tend to cause all readings to be displaced by a fixed amount, either too high or too low. Bias cannot be offset by taking more readings and averaging them. Mathematically Bias μ_e may be expressed as,

$$\mu_e = E(\hat{\theta}) - \theta \tag{5.1}$$

where $\hat{\theta}$ - represents an observed dimension

θ - is the true dimension of a specific unit

Imprecision:

Imprecision is the inability to repeat the results, when measurements are taken on the same unit. The dispersion of these measurements may be expressed as the standard deviation of the measurements. This type of error is usually normally distributed and is usually treated as independent of true dimension of the product. Imprecision σ_e can be defined as,

$$\sigma_e = [\ \mathrm{var}\ (\hat{\theta})]^{1/2} \tag{5.2}$$

5.1.3 Human Errors and Instrument Errors:

Measurement errors are due to combine effect of human and instrument errors. Human errors are caused by human

inaccuracies. The human errors can also be represented by bias (μ_h) and imprecision (σ_h). Instrument errors arise due to absolutely accurate. The instrument errors are characterized by bias μ_i and imprecision σ_i. The measurement error is assumed to be normally distributed and can be characterized by bias μ_e and imprecision σ_e which may be given by,

$$\mu_e = \mu_h + \mu_i \tag{5.3}$$

$$\sigma_e = (\sigma_h^2 + \sigma_i^2 + 2\rho\,\sigma_h\,\sigma_i)^{1/2} \tag{5.4}$$

where ρ is theoretical correlation coefficient between human error and instrument error.

5.2 Single Sampling Plan for Variables: (Single Specification Limit):

When product dimension, such as length, weight etc, are of concern there may be either an upper specification limit or lower specification limit or both. Considering the case of single, lower specification limit L, a product item is said to be acceptable if its dimension, x, is such that $x \geq L$, otherwise it is a defective.

Single sampling plan for variables is used to determine the acceptability of a lot. It is characterized by sample size, n, and acceptance constant k. A random sample of size n is taken and the appropriate dimension of each item

is measured. The measurements are averaged to constitute the statistic $\bar{x}_n$, which then is compared against a decision criterion $DC_{\bar{x}}$. For a single lower specification limit, the comparison is done as follows,

if $\bar{x}_n \geq DC_{\bar{x}} = L + k\sigma$ accept the lot reject otherwise

The lot standard deviation may be assumed to be known and constant. The decision variables n and k are determined to meet (AQL, α), (RQL, β) requirements.

The notations used in further analysis are summarized below:

- n - Sample size
- k - Acceptance constant
- L - Lower specification limit
- U - Upper specification limit
- p_1 - Acceptable quality level (AQL)
- p_2 - Reject quality level (RQL)
- p - General fraction defective
- Pa - Probability of acceptance
- x - True measurable dimension
- $f(x)$ - Distribution of x
- $\bar{x}_n$ - True sample average
- μ_p - True value of lot mean corresponding to fraction defective p
- σ - True standard deviation of lot
- x_e - Measurement error incurred in a single measurement

$f(x_e)$ - Distribution of the measurement error

μ_e - Bias

σ_e - Imprecision

Pa_e - Probability of acceptance under measurement error

x_o - Observed dimension

$f(x_o)$ - Observed lot distribution

μ_o - Observed or realized mean $(\mu_p + \mu_e)$

σ_o - Observed or realized standard deviation $= (\sigma^2 + \sigma_e^2)^{1/2}$

5.2.1 When Measurement Error is not Present:

The lot will be accepted when its sample average exceeds the decision criterion $DC_{\bar{x}}$. Thus probability of acceptance is given by [Duncan (5)]

$$Pa = \text{Prob.}\ (\bar{x}_n \geq DC_{\bar{x}} \mid p)$$

The above expression is for lower specification limit. An upper specification limit would require simple modification. Pa is then given by,

$$Pa = \text{Prob}\ (\bar{x}_n \geq L + k\sigma \mid p)$$

$$= \Pr\left(\frac{\bar{x}_n - \mu_p}{\sigma/\sqrt{n}} \geq \left(\frac{k\sigma + L - \mu_p}{\sigma} \right) \sqrt{n} \mid p \right) \quad (5.5)$$

Note that $\frac{\bar{x}_n - \mu_p}{\sigma/\sqrt{n}}$ is a standard normal deviate (Z_n) and $[(\mu_p - L)/\sigma]$ is a distributional deviate (Z_p). It gives number of standard deviations separating mean with lower

specification such that fraction defective equal to p results. Thus,

$$Pa = \text{Prob}\ (Z_n \geq [k - Z_p]\sqrt{n} \mid p) \tag{5.6}$$

the above equation (5.5) can be used to get two conditions for probability of acceptance at p_1 (AQL) and at p_2 (RQL) which can be solved to obtain the following [Duncan (5)]

$$n = \frac{(Z_\alpha + Z_\beta)^2}{(Z_{p_1} - Z_{p_2})^2} \tag{5.7}$$

$$\left.\begin{aligned} k_1 &= Z_{p_1} - Z_\alpha/\sqrt{n} \\ k_2 &= Z_{p_2} + Z_\beta/\sqrt{n} \end{aligned}\right\} \quad k = \frac{k_1 + k_2}{2} \tag{5.8}$$

Sample size is rounded up or down to an integer giving two different value for acceptance constant k_1 and k_2 and the mean of two is taken as the acceptance constant.

5.2.2 <u>When Measurement Error is Present</u>:

Each measurement will involve some deviation from true value of dimension. This deviation x_e is a random variable assumed to be normally distributed and characterized by bias and imprecision which are assumed to be known. The lot distribution and error distribution are assumed to be independent.

When measurement error is present distribution of the observed dimension is convolution of the lot distribution and the measurement error distribution. That is,

$$f_3\,(x_o = x + x_e) \;=\; f_1(x)\,.\,f_2(x_e) \qquad (5.9)$$

The observed distribution has mean and standard deviation given by,

$$\mu_o \;=\; \mu_p + \mu_e \qquad (5.10)$$

$$\sigma_o \;=\; (\sigma^2 + \sigma_e{}^2)^{1/2} \qquad (5.11)$$

$$\sigma_o \;=\; \sigma(1 + \frac{\sigma_e{}^2}{\sigma^2})^{1/2}$$

$$=\; (1 + \frac{1}{h})^{1/2}$$

$$=\; \sqrt{\frac{h+1}{h}} \qquad (5.12)$$

where $h = \frac{\sigma^2}{\sigma_e{}^2}$, ratio of lot distribution variance to error variance.

5.2.3 Effect of Measurement Error on OC Curve:

Case 1: When only bias is present.

We had seen before probability of acceptance in absence of error is given by Eq. (5.12) as,

$$Pa \;=\; \text{Prob}\;(\frac{\bar{x}_n - \mu_p}{\sigma/\sqrt{n}} \geq [\frac{k\,\sigma + L - \mu_p}{\sigma}]\sqrt{n} \mid p)$$

Bias will cause $\bar{x}_n$ to be replaced by $\bar{x}_o$ and μ_p by μ_o in the above equation.

$$Pa_e \;=\; \text{Prob}\;(\frac{\bar{x}_o - \mu_o}{\sigma/\sqrt{n}} \geq [\frac{k\,\sigma + L - \mu_o}{\sigma}]\sqrt{n} \mid p) \qquad (5.13)$$

From the above equation it is possible to say,

if $\mu_e < 0,\; Pa_e < Pa$

$\mu_e > 0,\; Pa_e > Pa$

That is if there is tendency to underestimate the actual dimension, an observed dimension closer to lower specification limit is realized and hence probability of acceptance is diminished and if there is a tendency to over estimate the actual dimension, probability of lot acceptance is increased. Thus we see bias has a translation effect on the OC curve.

Case II: When Only Imprecision is Present:

In the presence of only imprecision, the observed probability of acceptance will be given by,

$$Pa_e = \text{Prob} \left(\frac{\bar{x}_o - \mu_p}{\sigma_o/\sqrt{n}} \geq \left[\frac{k\sigma + L - \mu_p}{\sigma_o} \right] \sqrt{n} \mid p \right)$$

$$= \text{Prob} \left(\frac{\bar{x}_o - \mu_p}{\sigma_o/\sqrt{n}} \geq \left[\frac{k\sigma + L - \mu_p}{\sigma\sqrt{\frac{h+1}{h}}} \right] \sqrt{n} \mid p \right)$$

$$= \text{Prob} \left(Z_n \geq \left[\frac{k\sigma + L - \mu_p}{\sigma_o} \right] \sqrt{n} \sqrt{\frac{h}{h+1}} \mid p \right) \tag{5.14}$$

It can be seen that $\bar{x}_n$ is replaced by $\bar{x}_o$ and σ in the denominator on both sides are replaced by σ_o, σ in the numerator on right hand side is kept as such because $k\sigma + L$ is the value of $DC_{\bar{x}}$, which is supplied to the inspector. This value remains specified to the inspector in advance. The effect of imprecision can be studied by recognizing $\sqrt{\frac{h}{h+1}}$ will be in the range,

$$0 \leq \sqrt{\frac{h}{h+1}} < 1$$

If $\left[\frac{k\sigma + L - \mu_p}{\sigma}\right]\sqrt{n} < 0$, $Pa_e < Pa$, and

If $\left[\frac{k\sigma + L - \mu_p}{\sigma}\right]\sqrt{n} > 0$, $Pa_e > Pa$.

That is, for a fraction defective where desired probability of acceptance is greater than 0.50 the actual probability of acceptance will lie in the range $0.50 < Pa_e \leq Pa$. Conversely at a fraction defective where desired probability is less than 0.50, $Pa \leq Pa_e < 0.5$. In short imprecision tends to rotate or flatten the OC curve and makes the plan less discriminating between good and bad lots.

Case III: When both bias and imprecision are present.

In presence of both bias and imprecision observed probability of acceptance is given by,

$$Pa_e = \text{Prob}\left(\frac{\bar{x}_o - \mu_o}{\sigma_o/\sqrt{n}} \geq \left[\frac{k\sigma + L - \mu_o}{\sigma_o}\right]\sqrt{n} \mid p\right)$$

$$= \text{Prob}\left(Z_n \geq \left[\frac{k\sigma + L - \mu_p - \mu_e}{\sigma}\right]\sqrt{n}\sqrt{\frac{h}{h+1}} \mid p\right) \tag{5.15}$$

The effect of measurement errors on probability of acceptance will then be depend upon the relative values of the bias and imprecision. Bias is a dominating member and hence the effect will follow pattern similar to that due to

bias only. Numerical example in later section would make the concepts clear.

5.2.4 A Numerical Example:

Let us consider a typical example [Duncan (5)], where $L = 17000$ psi, standard deviation $\sigma = 800$ psi,

$$p_1(\text{AQL}) = 0.01$$
$$p_2(\text{RQ}) = 0.08$$
$$\alpha = 0.05$$
$$\beta = 0.10$$

We have for above plan,

$$Z_\alpha = 1.6449, \quad Z_\beta = 1.2816, \quad Z_{p_1} = 2.3262,$$
$$Z_{p_2} = 1.4053.$$

$$n = \left(\frac{1.6449 + 1.2816}{2.3263 - 1.4053} \right)^2$$
$$= 10.0967$$
$$k = 2.3263 - \frac{1.6449}{10.0967}$$
$$= 1.8086$$

For the plan found above, i.e. $n = 10.0967$, $k = 1.8086$ along with $L = 17000$, $\sigma = 800$ let us try to examine the effect of bias and imprecision by finding OC curve for various values of bias and imprecision. We will not round up or down n obtained before to make it an integer so as to have an idea of the effect of measurement errors. Table 29 gives probability

of acceptance for few selected incoming quality levels and for measurement error distributions. The Fig. 14 shows the effect of measurement errors on the OC curve. Results obtained in Table 29 as well as in the Fig. 14 are in accordance with the discussion presented in Sec. 5.2.3.

5.2.5 Upper Specification Limit:

If the decision is to be based for an upper specification limit, U, then the OC curve in presence of bias and imprecision both, can be expressed as,

$$Pa = \text{Prob}\left(\frac{\bar{x}_n - \mu}{\sigma/\sqrt{n}} \leq \left[\frac{-k\sigma + U - \mu}{\sigma} \right] \sqrt{n} \mid p \right) \tag{5.16}$$

$$Pa_e = \text{Prob}\left(\frac{\bar{x}_o - \mu_o}{\sigma_o/\sqrt{n}} \leq \left[\frac{-k\sigma + U - \mu_o}{\sigma_o} \right] \sqrt{n} \mid p \right) \tag{5.17}$$

Substituting $\mu_o = \mu_p + \mu_e$, $\sigma_o = \sigma\sqrt{\frac{h+1}{h}}$ and simplifying,

$$Pa_e = \text{Prob}\left(Z_n \leq \left[-k - \frac{\mu_e}{\sigma} + Z_p\right] \sqrt{n} \sqrt{\frac{h}{h+1}} \mid p \right) \tag{5.18}$$

The effect of imprecision, as it can be seen from the Eq. (5.18), will be the same as in the case of lower specification limit. However, the effect of bias will be just the opposite.

5.2.6 Design of Plan Compensating for Measurement Errors:

Case 1: Lower Specification Limit:

Probability of acceptance when both bias and imprecision are present is given by Eq. (5.15), which can be simplified to,

Table 29: Probability of Acceptance in Presence of Measurement Errors (Single Sampling Plan for Variables).

μ_p	$\mu_e = 0$ $\sigma_e = 0$	$\mu_e = 0$ $\sigma_e = 200$	$\mu_e = 0$ $\sigma_e = 400$	$\mu_e = 200$ $\sigma_e = 0$	$\mu_e = 100$ $\sigma_e = 0$	$\mu_e = -100$ $\sigma_e = 0$	$\mu_e = 100$ $\sigma_e = 200$	$\mu_e = 200$ $\sigma_e = 200$
0.005	0.9926	0.9910	0.9854	0.9994	0.9977	0.9793	0.9975	0.9992
0.010	0.9500	0.9447	0.9293	0.9927	0.9793	0.8938	0.9786	0.9910
0.020	0.7823	0.7752	0.7561	0.9422	0.8804	0.6489	0.8786	0.9366
0.030	0.5898	0.5871	0.5805	0.8462	0.7338	0.4325	0.7321	0.8391
0.040	0.4263	0.4283	0.4349	0.6978	0.5836	0.2799	0.5830	0.7224
0.050	0.3015	0.3070	0.3210	0.6081	0.4492	0.1799	0.4515	0.6650
0.060	0.2101	0.2172	0.2351	0.4954	0.3415	0.1142	0.3426	0.4956
0.070	0.1461	0.1524	0.1721	0.3961	0.2544	0.0728	0.2560	0.3991
0.080	0.1000	0.1063	0.1257	0.3128	0.1880	0.0465	0.1899	0.3180
0.090	0.0682	0.0742	0.0915	0.2437	0.1375	0.0297	0.1394	0.2502
0.100	0.0472	0.0523	0.0672	0.1897	0.1009	0.0224	0.1027	0.1970
0.110	0.0321	0.0363	0.0480	0.1455	0.0731	0.0123	0.0747	0.1529
0.120	0.0220	0.0254	0.0359	0.1112	0.0530	0.0080	0.0544	0.1185

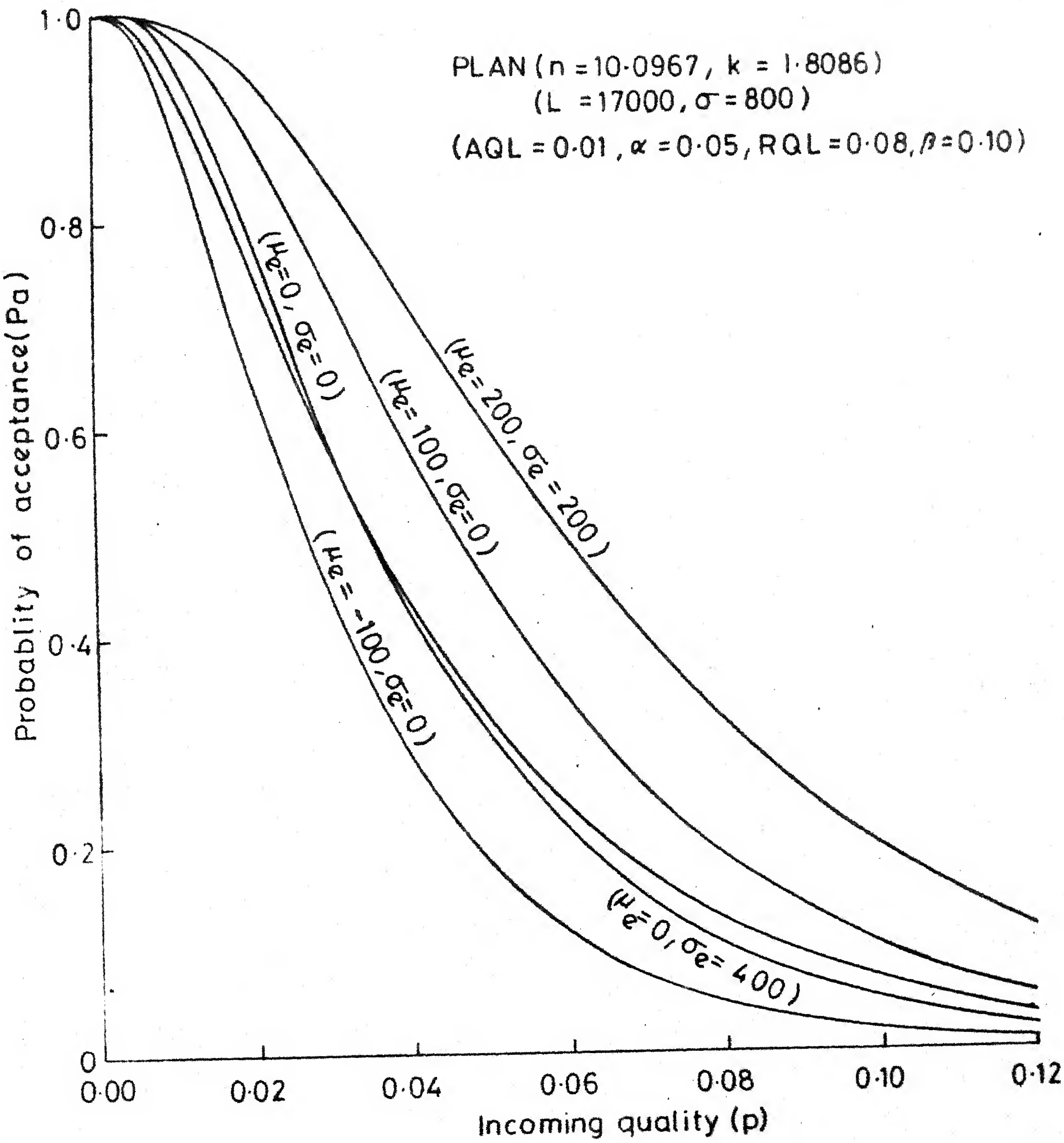

Fig 14 OC Curves in presence of errors (Variable sampling plan single specification limit)

$$Pa_e = \text{Prob}\ (Z_n \geq [k - \frac{\mu_e}{\sigma} + Z_p]\sqrt{n}\sqrt{\frac{h}{h+1}} \mid p) \quad (5.19)$$

For $p = p_1$ requirement is $Pa_e = 1 - \alpha$, and

for $p = p_2$ requirement is $Pa_e = \beta$.

Substituting these requirements in Eq. (5.19) and simplifying, we get,

$$Z_{1-\alpha} = (k - \frac{\mu_e}{\sigma} - Z_{p_1})\sqrt{n}\sqrt{\frac{h}{h+1}} \quad (5.20)$$

$$Z_\beta = (k - \frac{\mu_e}{\sigma} - Z_{p_2})\sqrt{n}\sqrt{\frac{h}{h+1}} \quad (5.21)$$

Let, $k' = k - \mu_e/\sigma$, $n' = \frac{n\ h}{h+1}$

$$(k' - Z_{p_2})\sqrt{n'} = Z_{1-\alpha} \quad (5.22)$$

$$(k' - Z_{p_2})\sqrt{n'} = Z_\beta \quad (5.23)$$

Solving as in case of no measurement error, we get [Duncan (5)]

$$n' = \frac{(Z_\alpha + Z_\beta)^2}{(Z_{p_1} + Z_{p_2})^2} \quad (5.24)$$

$$\left.\begin{aligned} k_1' &= Z_{p_1} - \frac{Z_\alpha}{\sqrt{n'}} \\ k_2' &= Z_{p_2} + \frac{Z_\beta}{\sqrt{n'}} \end{aligned}\right\} k' = \frac{k_1' + k_2'}{2} \quad (5.25)$$

Knowing n' and k' we can get sample size and acceptance constant as,

$$n = \sqrt{\frac{h+1}{h}}\ \frac{(Z_\alpha + Z_\beta)^2}{(Z_{p_1} - Z_{p_2})^2} \quad (5.26)$$

$$k = k' + \frac{\mu_e}{\sigma} \quad (5.27)$$

Here n' and k' are same as n and k when analysis is done without measurement errors. Hence it can be seen bias affects acceptance constant and imprecision affects the sample size. Both together change the plan. Acceptance constant increases with positive bias ($\mu_e > 0$) because positive bias will lead to too high values, which would have been otherwise accepted. Conversely acceptance constant decreases with negative bias. Sample size increases in presence of imprecision. The presence of imprecision would introduce additional variability and hence would lead to increased sample size. The observed OC curve (Pa_e) completely overlaps desired OC curve (Pa).

<u>Case 2</u>: Upper specification limit.

When upper specification limit is the criterion probability of acceptance is given by Eq. (5.17),

$$Pa_e = \text{Prob}\left(\frac{\bar{x}_o - \mu_o}{\sigma_o/\sqrt{n}} \leq \left[\frac{-k\sigma + U - \mu_o}{\sigma_o} \right]\sqrt{n} \mid p \right)$$

The above equation can be used in the same way as in case of lower specification limit to give,

$$\text{Sample size } n = \sqrt{\frac{h+1}{h}} \left(\frac{Z_\alpha + Z_\beta}{Z_{p_1} - Z_{p_2}} \right)^2 \qquad (5.28)$$

$$\text{Acceptance constant } k = k' - \frac{\mu_e}{\sigma} \qquad (5.29)$$

where k' is the acceptance constant for the case when there is no measurement error (same as the case of lower specification limit).

Again sample size increases in presence of imprecision. However, bias has an opposite effect on the acceptance constant, which is even obvious intuitively. The observed OC curve can thus be made to overlap the desired OC curve, (and hence the term compensating design).

5.3 Variables Sampling Plan with Double Specification Limits:

Case I: When error free measurement is assumed:

The minimum fraction defective of the lot or process with upper and lower specification limits occurs when the process mean falls exactly between the specification limits. In this case Z deviate at each end will be,

$$\frac{U - \mu}{\sigma} = \frac{\mu - L}{\sigma} = \frac{U - L}{2\sigma}$$

and the proportion of area beyond $\pm$ Z will be the fraction defective. When there are two specification limits, therefore, the first step is to check whether area beyond $Z = \pm \frac{U - L}{2\sigma}$ is greater than what is acceptable. If so, we may reject the lot without sampling.

If, however, the minimum possible fraction defective is better than what we seek, it may pay to sample, since may be either favorably or unfavorably located. Let us consider a case, in which, if process is centred, there will be practically no defective material. This will occur if $\frac{U - L}{2\sigma} \geq 3$. For such widely spread limits there will be two single limit plans, one for each end.

The third case can be when two limits are not relatively far apart, say $\frac{U - L}{2\sigma} < 3$. But still are not so close that minimum fraction defective to be obtained, if is centred between two limits, is less than the acceptable level. Then the procedure is to find by trial and error a suitable which will give fraction defective equal to acceptable quality level and then design the plan [Duncan (5)].

Case II: With Measurement are not Error-free:

When the measurement errors are present, the sample mean $\bar{x}_n$ is replaced by realized sample mean $\bar{x}_o$. σ is replaced by σ_o. The decision criterion given to inspector contains σ, but σ in $DC_{\bar{x}}$ is not replaced by σ_o because it has been already provided to erroneous inspector using erratic measuring instruments.

The three steps still remain the same as in case of error-free measurements. The first step is checking, whether acceptable quality level can be obtained, by finding minimum possible fraction defective. This will be obtained when observed process mean μ_o will fall, exactly between upper and lower specification limits. Minimum fraction defective will be given by the area beyond $\pm$ Z of normal curve where $Z = \frac{U - L}{2\sigma_o}$.

The second case is when $\frac{U - L}{2} > 3\sigma_o$, and in this case two single limit plan in presence of measurement errors can be found as described in Sec. 5.2.

The third case will be when minimum fraction defective is less than acceptable quality level but $\frac{U - L}{2\sigma} \not> 3\sigma_0$. In such a case we resort to same trial and error procedure of finding appropriate setting to get the fraction defective equal to acceptable quality level. Then steps to find n and k will proceed on the same lines as in the case of single specification limit. Thus we see in the case of double specification limit there is not much change as compared to the procedure for single specification limit.

5.4 Sequential Sampling Plans for Variables (Single Sided Alternative):

When quality characteristic is measured and question is whether a standard has exceeded (or fallen short of) the mean of normal distribution (with known standard deviation). The sequential sampling plan is based on Σ x can be effectively used. Since $\bar{x} = \Sigma x/n$, a simple sampling plan based on Σ x is equivalent to a plan based on $\bar{x}$.

When a quality characteristic is normally distributed and when the standard deviation is known, a sequential probability ratio plan based on Σ x takes the following form. A unit is taken from the lot and a measurement x is made of the given quality characteristic. The result is plotted on a chart shown on the next page. If the value falls above AB the lot is accepted. If the value falls below CD the lot is rejected. If the value falls between the two parallel lines,

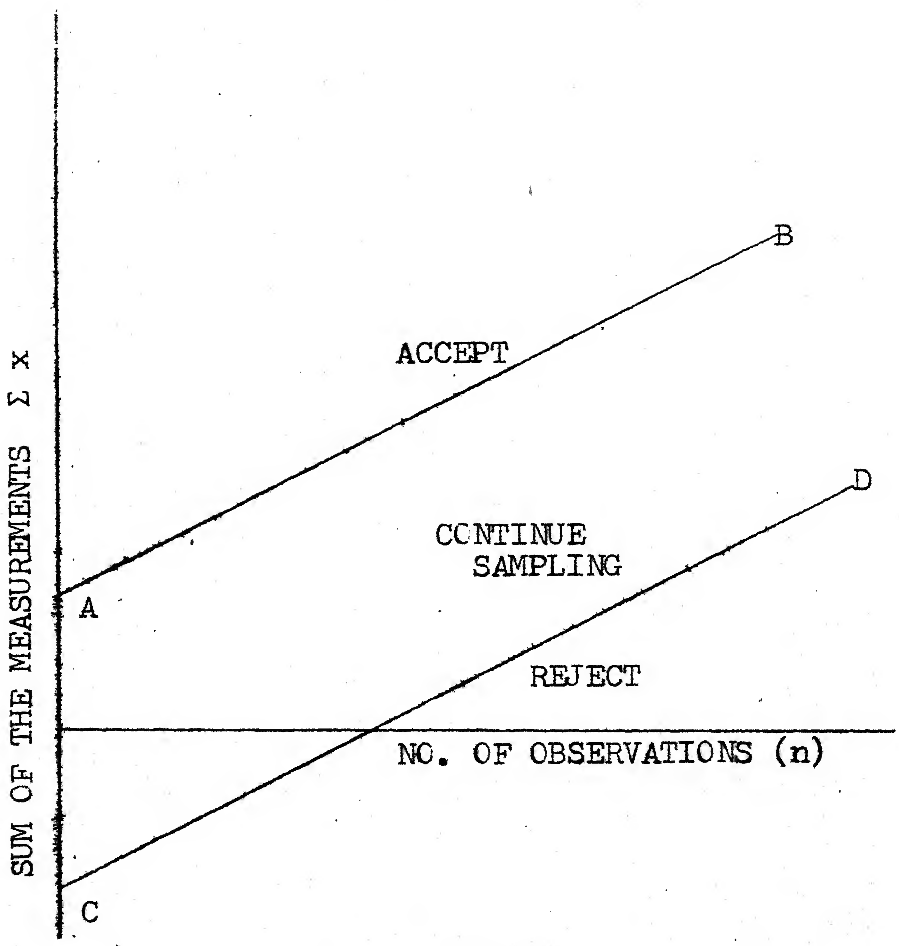

Fig. 15: Sequential Sampling Plan for Variables.

another item is taken and x is measured. Σx is found and plotted on the chart and checked for the three actions. It is possible to have acceptance and rejection lines to be just opposite (upper line is rejection line and lower line is acceptance line) if low values of x are derived.

5.4.1 SPR Plan Based on Specified Values of μ_1, μ_2, α and β:

For specified values of μ_1, μ_2, α and β, and a given the equation for the two parallel lines are given for case of single sided alternative.

$$T = h_2 + s\,n \tag{5.30}$$

$$T = -h_1 + s\,n \tag{5.31}$$

where for lower μ_2 (Analogous to lower specification limit)	for an upper μ_2 (Analogous to upper specification limit)	
$h_1 = \dfrac{b\,\sigma^2}{\mu_1-\mu_2}$	$h_1 = \dfrac{b\,\sigma^2}{\mu_2-\mu_1}$	(5.32)
$h_2 = \dfrac{a\,\sigma^2}{\mu_1-\mu_2}$	$h_2 = \dfrac{a\,\sigma^2}{\mu_2-\mu_1}$	(5.33)
$s = \dfrac{\mu_1+\mu_2}{2}$	$s = \dfrac{\mu_1+\mu_2}{2}$	(5.34)
$a = \log_e\left(\dfrac{1-\alpha}{\beta}\right)$	$a = \log_e\left(\dfrac{1-\beta}{\alpha}\right)$	(5.35)
$b = \log_e\left(\dfrac{1-\beta}{\alpha}\right)$	$b = \log_e\left(\dfrac{1-\alpha}{\beta}\right)$	(5.36)

It may be noted that the spread between the lines but not the slope depends upon σ.

5.4.2 Effect of Measurement Error on Probability of Acceptance:

The probability of acceptance for a SPR plan based on Σ x with a single μ_2 is given by following formulae.

$$K = \frac{2(s-\mu)}{\sigma^2} \qquad K_e = \frac{2(s-\mu-\mu_e)}{(\sigma^2+\sigma_e^2)} \tag{5.37}$$

$$t_1 = h_1 K \qquad t_{1e} = h_1 K_e$$

$$t_2 = (h_1 + h_2) K \qquad t_{2e} = (h_1 + h_2) K_e \tag{5.38}$$

$$Pa = \frac{e^{t_1} - 1}{e^{t_2} - 1} \qquad Pa_e = \frac{e^{t_{1e}} - 1}{e^{t_{2e}} - 1} \tag{5.39}$$

Without error With errors

To illustrate the effect of bias and imprecision on the OC curve, a numerical example is considered.

($\mu_1 = 0.1350$, $\mu_2 = 0.1300$, $\alpha = 0.05$,

$\beta = 0.10$, and $\sigma = 0.006$) [Duncan (5)]

This gives the plan in absence of errors as,

$h_1 = 0.0208$, $h_2 = 0.0162$, $s = 0.1325$

For the above plan and for few representative values of bias and imprecision, probability of acceptance was computed and is given in the Table 30. The figure 16 shows the OC curve in presence of measurement errors. The effect of bias and imprecision on the OC curve is same as in the case

Table 30: Probability of Acceptance in Presence of Measurement Errors (Sequential Sampling Plan for Variables).

μ_p \ μ_e	0.00	0.0005	0.001	-0.001	0.000	0.000	0.001	0.002
σ_e	0.00	0.00	0.00	0.00	0.003	0.004	0.004	0.004
0.1200	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0008	0.0015
0.1210	0.0000	0.0000	0.0001	0.0000	0.0003	0.0008	0.0015	0.0027
0.1220	0.0001	0.0001	0.0002	0.0000	0.0005	0.0015	0.0027	0.0051
0.1230	0.0002	0.0003	0.0005	0.0001	0.0011	0.0027	0.0051	0.0094
0.1240	0.0005	0.0007	0.0012	0.0002	0.0022	0.0051	0.0094	0.0173
0.1250	0.0012	0.0019	0.0029	0.0005	0.0045	0.0094	0.0173	0.0323
0.1260	0.0029	0.0045	0.0071	0.0012	0.0094	0.0173	0.0323	0.0590
0.1270	0.0071	0.0112	0.0176	0.0029	0.0190	0.0323	0.0590	0.1077
0.1280	0.0176	0.0271	0.0424	0.0071	0.0391	0.0590	0.1077	0.1877
0.1290	0.0424	0.0658	0.1014	0.0176	0.0781	0.1077	0.1877	0.3138
0.1300	0.1014	0.1515	0.2245	0.0424	0.1515	0.1877	0.3138	0.4752
0.1310	0.2245	0.3215	0.4391	0.1014	0.2806	0.3138	0.4752	0.6466
0.1320	0.4391	0.5621	0.6802	0.2245	0.4616	0.4752	0.6466	0.7904
0.1330	0.6802	0.7840	0.8621	0.4391	0.6594	0.6466	0.7904	0.8898
0.1340	0.8621	0.9158	0.9491	0.6802	0.8178	0.7904	0.8898	0.9451
0.1350	0.9491	0.9705	0.9831	0.8621	0.9158	0.8898	0.9451	0.9743
0.1360	0.9831	0.9904	0.9946	0.9491	0.9634	0.9451	0.9743	0.9880
0.1370	0.9946	0.9969	0.9983	0.9831	0.9850	0.9743	0.9880	0.9946
0.1380	0.9983	0.9990	0.9995	0.9946	0.9939	0.9880	0.9946	0.9975
h_{1e}	0.0208	0.0208	0.0208	0.0208	0.0260	0.0301	0.0301	0.0301
h_{2e}	0.0162	0.0162	0.0162	0.0162	0.0203	0.0234	0.0234	0.0234
s_e	0.1325	0.1330	0.1335	0.13[illegible]5	0.1325	0.1325	0.1335	0.1345

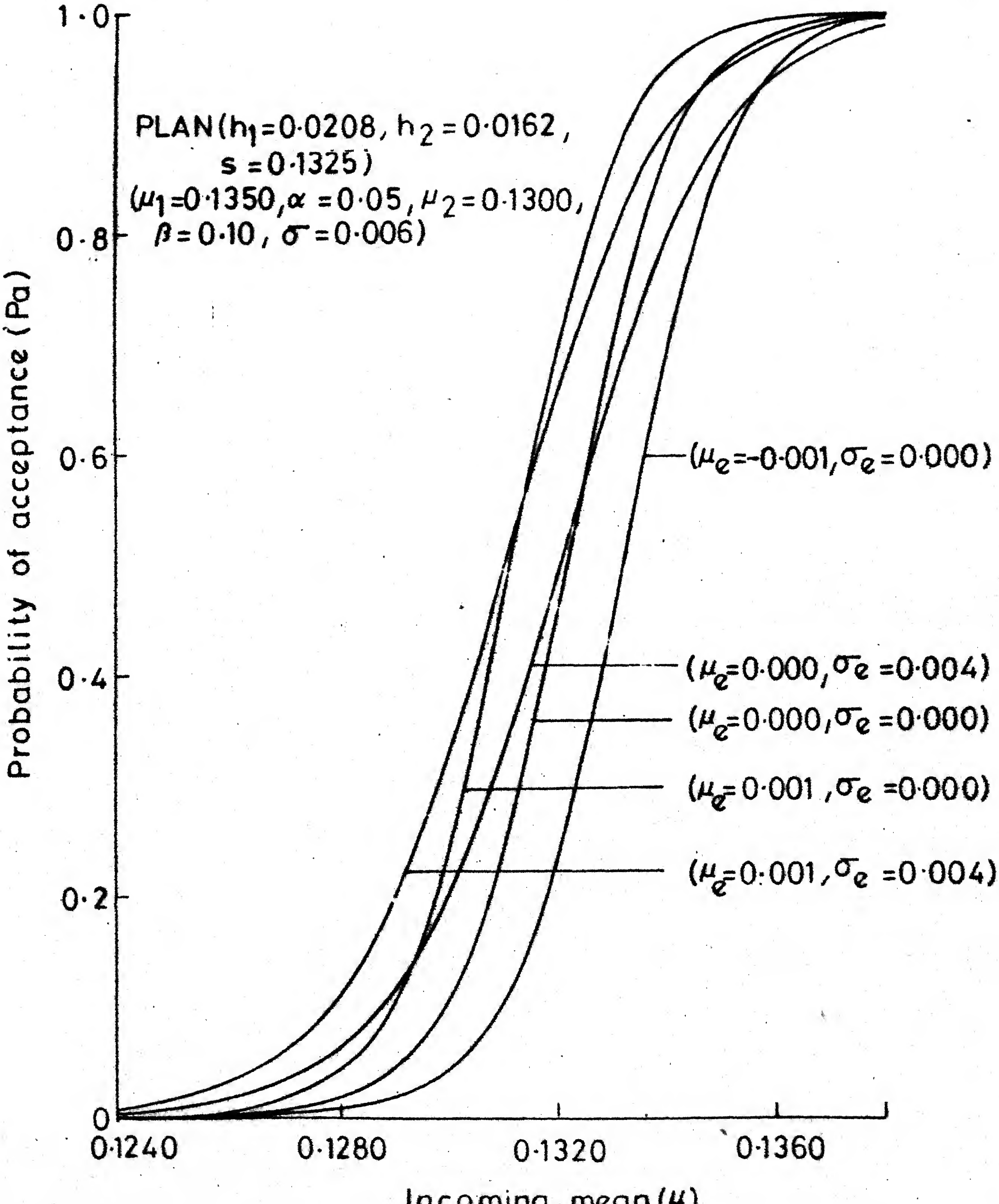

Fig.16 OC Curve in presence of errors (Sequential sampling plan for variables with single specification limit)

of single sampling plan for variables. Bias shifts the OC curve (or translates it) in either direction depending upon whether bias is positive or negative. Imprecision flattens the OC curve to make the plan less discriminating between good and bad lots.

5.4.3 Effect of Measurement Errors on Average Sampling Number:

The average sample number curve for a SPR plan based on Σ x can be derived from the following formula when bias and imprecision are not present [SRG report (13)]

$$ASN = \frac{Pa\ (h_1 + h_2) - h_2}{\mu - s}$$

In presence of μ_e and σ_e we will have Pa_e instead of Pa as discussed before. ASN will be modified to,

$$ASN_e = \frac{Pa_e\ (h_1 + h_2) - h_2}{\mu + \mu_e - s}$$

Again for same sequential plan considered in Sec. (5.4.3). ASN_e is computed for a few representative pairs of bias μ_e and imprecision σ_e. The results are given in Table 31. The Fig. 17 shows the effect of bias and imprecision on the ASN curve.

Bias when present shifts the OC curve in either direction depending upon its sign. ASN is basically dependent upon probability of acceptance and probability of rejection. Both of which are complement of each other. ASN increases in the

Table 31: Average Sample Number in Presence of Measurement Errors (Sequential Sampling Plan for Variables).

μ_p \ μ_e / σ_e	0.00 / 0.00	0.0005 / 0.000	0.001 / 0.000	-0.001 / 0.000	0.000 / 0.003	0.000 / 0.004	0.001 / 0.004	0.002 / 0.004
0.1200	1.665	1.734	1.810	1.542	1.664	1.664	1.807	1.977
0.1210	1.810	1.892	1.982	1.665	1.809	1.807	1.977	2.180
0.1220	1.982	2.081	2.190	1.810	1.980	1.977	2.180	2.426
0.1230	2.190	2.311	2.446	1.982	2.186	2.180	2.426	2.729
0.1240	2.446	2.598	2.769	2.190	2.439	2.426	2.729	3.103
0.1250	2.769	2.963	3.185	2.446	2.752	2.729	3.103	3.566
0.1260	3.185	3.441	3.736	2.769	3.148	3.103	3.566	4.159
0.1270	3.736	4.079	4.480	3.185	3.656	3.566	4.139	4.807
0.1280	4.480	4.952	5.498	3.736	4.303	4.139	4.807	5.545
0.1290	5.498	6.125	6.832	4.480	5.120	4.807	5.545	6.130
0.1300	6.823	7.601	8.332	5.498	6.081	5.545	6.130	6.437
0.1310	8.332	8.910	9.109	6.823	6.949	6.130	6.437	6.253
0.1320	9.109	8.910	8.742	8.332	7.445	6.437	6.253	5.634
0.1330	8.742	8.212	7.404	9.109	7.199	6.253	5.634	4.852
0.1340	7.402	6.546	5.731	8.742	6.310	5.634	4.852	4.051
0.1350	5.731	5.039	4.452	7.404	5.237	4.852	4.051	3.391
0.1360	4.452	3.964	3.558	5.731	4.244	4.051	3.391	2.866
0.1370	3.558	3.219	2.935	4.452	3.479	3.391	2.866	2.463
0.1380	2.935	2.696	2.491	3.558	2.906	2.866	2.463	2.149

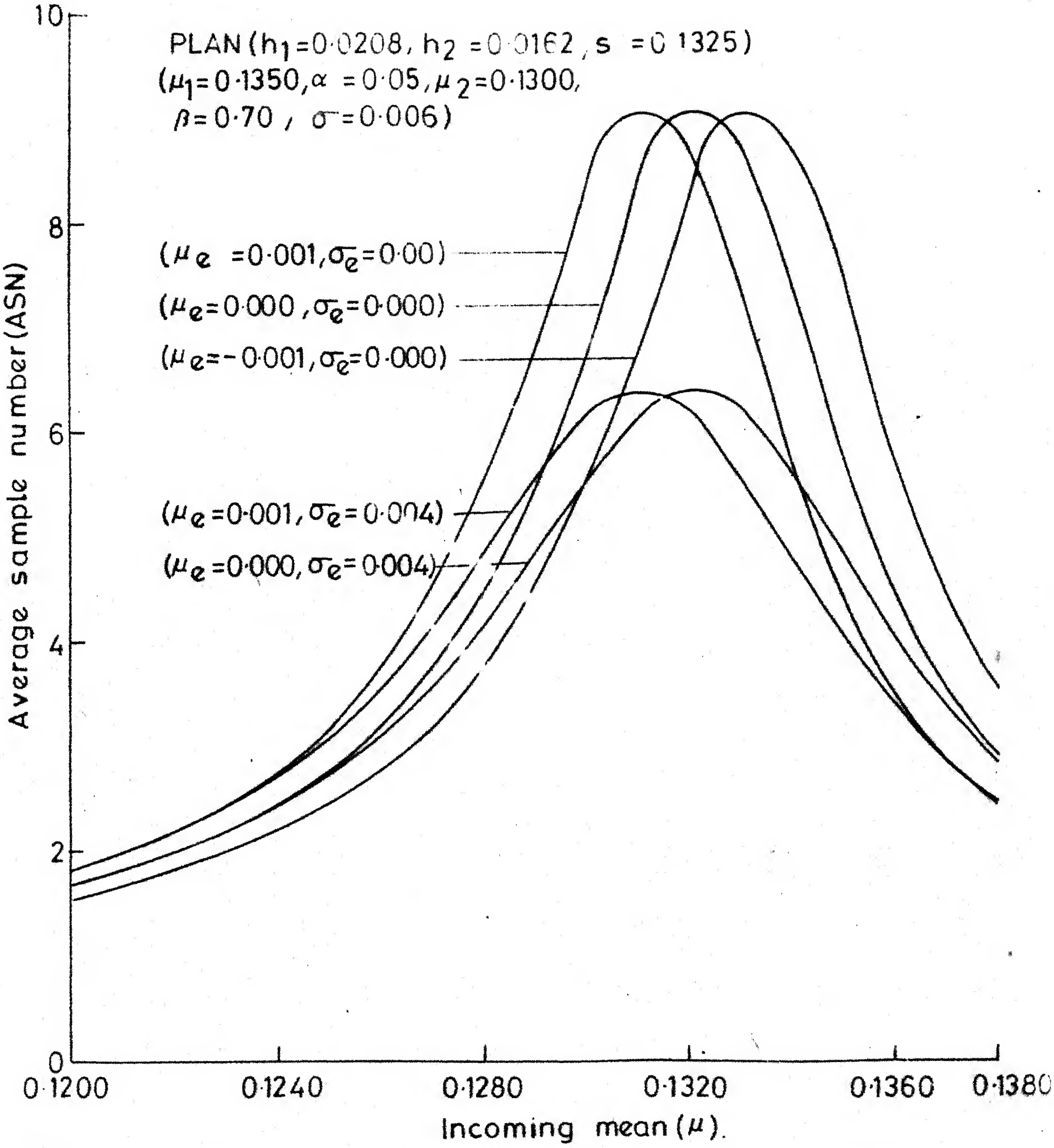

Fig 17 Average sampling number in presence of errors (Sequential plan for variables with single specification limit)

presence of imprecision for some part of the curve and then it decreases as can be seen from the figure 17.

5.4.4 <u>Compensating Sequential Plan in Presence of Measurement Errors:</u>

We have seen the effect of measurement errors on probability of acceptance in the Sec. (5.4.3). The plan based on μ_1, μ_2, α and β requirements is to be designed when measurement errors are present.

The development follows on exactly similar lines as in case of sequential sampling plan for attributes and single sampling plan for variables. The following are the decision variables of the compensating plan.

$$h_{1e} = \frac{b\,(\sigma^2 + \sigma_e^{\,2})}{\mu_2 - \mu_1} \tag{5.42}$$

$$h_{2e} = \frac{a\,(\sigma^2 + \sigma_e^{\,2})}{\mu_2 - \mu_1} \tag{5.43}$$

$$s_e = \frac{\mu_1 + \mu_2}{2} + \mu_e \tag{5.44}$$

The results can be obtained by replacing μ by $(\mu+\mu_e)$ and σ^2 by $(\sigma^2 + \sigma_e^{\,2})$ in equations for the case when measurements are error free. For the equivalent plan given by h_{1e}, h_{2e} and s_e, it can be verified that observed OC curve (Pa_e) will completely overlap the desired OC curve (Pa)(given in the Appendix B). That is if we know measurement errors (μ_e, σ_e) and the plan variables h_1, h_2 and s and also (the standard

deviation of the process), we can have the compensating plan given by Eqs. (5.42), (5.43) and (5.44).

5.5 Economic Effect of Measurement Errors on Variables Sampling Plan:

The economic effects of measurement errors on variables sampling plan have been discussed by Case et al (35). They have considered the case of double specification limit. The inspection cost and cost of accepting a defective and cost of rejecting an entire lot are considered. The effect of bias in absence of imprecision is dramatic. A bias which is 33.3 percent of six standard deviation natural tolerance spread increases the cost to nearly 600 percent. The increase however depends upon cost coefficients and other parameters. Similarly the imprecision equal to 66.7 percent of six standard deviations increases cost by 150 percent. The effect of imprecision is thus less dramatic, but still, it cannot be ignored.

5.6 Conclusions:

The following are conclusions based on the treatment in the present chapter.

1. The measurement errors are represented in form of bias (μ_e) and imprecision (σ_e).

2. The measurement errors are due to combined effect of human errors and instrument errors,

$$\mu_e = \mu_h + \mu_i$$

$$\sigma_e = (\mu_h^2 + \mu_i^2 + 2\rho\sigma_h \sigma_i)$$

3. The presence of bias and imprecision significantly affect the OC curve. But the design based on AQL, α, RQL and β can be modified to account for measurement errors, so as to obtained desired OC curve.

4. The effect on the SPR based plan is similar to that on plans based on single or double specification limit.

5. The economic effects of the measurement errors are too drastic to be ignored. But further analysis is required to design a plan based on economic model and then account for the presence of measurement errors.

6. The analysis of SPR based plan with two sided alternative and analysis of plans when standard deviation is unknown can be of interest for further research.

CHAPTER VI

CONCLUSIONS AND SCOPE FOR FURTHER WORK

6.1 Conclusions:

Quality control is a major activity in the production system. The objective of quality control system is to satisfy the quality needs of the specific customers in the most economic manner possible. The effectiveness of the quality control system in meeting the quality objectives depends largely on how well people do their jobs. Job performance is a function of the particular set of individual, physical and organizational factors that influence a person in his job. The general term for these influences is human factors.

Human inspector using his basic senses to detect faults is the major element of the quality control system. The evidence from the studies done in the past and the one at hand gives strong grounds for claiming dominant position of human inspection in quality control system. A well designed quality control system would try to accrue the advantage a human inspector can offer but it has to make allowances for his inherent limitations. The inspector is rarely perfect either in detection or in diagnosis.

In the context of acceptance sampling by attributes an inspector may classify a good items as defective (type I error)

or a defective as good (type II error). In case of variables being measured on a continuous scale the inspection error together with instrument error give rise to what may be termed as measurement error. The measurement errors are characterized by bias and imprecision.

The effect of inspection errors on various performance measures is found to be significant. The performance measures such as probability of acceptance average outgoing quality etc. are studied in the presence of inspection errors. It is found that the nature of the effect of inspection errors on the performance measures for single, double and sequential sampling plan is not significantly different.

Regarding the operating characteristic curve of a sampling plan, it is found that type I error reduces the probability of acceptance and type II error increases the probability of acceptance. It is so, because the type I error incorrectly classifies good items as defectives and type II errors classifies defective items as good.

Single sampling plan consists of a fixed sample size. Hence Average Sample Number (ASN) is constant for a single sampling plan. For double and sequential sampling plan ASN is a function of incoming quality. The effect of type I error is to increase ASN until p is better than certain level (Say p_{N_1}) beyond which the effect is to decrease ASN. For type II error, ASN decreases until certain incoming quality

(p_{N_2}) beyond which it increases in the presence of error. When both errors are present the net effect depends upon relative values of e_1 and e_2. But e_1 being dominating member the effect is usually similar to that in presence of e_1. The above analysis has been presented in Chapter IV.

Average outgoing quality (AOQ) is also significantly affected in presence of inspection errors. Type I error decreases AOQ for all incoming quality levels because of more screening inspection (Lots are rejected more often). In presence of type II errors AOQ increases for all incoming quality levels because of erroneous classification and acceptance of defectives as good. The shape of AOQ curve also changes. At higher values of p where Pa tends to be negligibly small, AOQ curve starts rising because of increased number defectives being classified as good. In case of some typical combination of type I and type II errors AOQ curve may not show any decreasing trend throughout the range of incoming quality. In the range where p is small e_1 and e_2 both have AOQ with an increasing trend and after that when it should decrease (at higher value of p), e_2 is dominating and hence the curve keeps rising.

Average total inspection (ATI) increases in presence of type I errors due to decrease in the probability of acceptance and increased inspection of rejected lots. The effect of type II errors is just the opposite.

A model has been presented for the economic analysis of double sampling plan in the presence of errors. The computational results could not be obtained because of shortage of time.

The effect of measurement errors on the variables sampling plans is also studied. Measurement errors are characterized by bias and imprecision. The effect of measurement errors on probability of acceptance is studied. Compensating design procedures to account for measurement errors have also been discussed.

In the presence of bias the OC curve translates in either direction. For a plan designed for the lower specification limit, a positive bias would shift the OC curve towards right because it would increase the measurements and lead to higher probability of acceptance. Negative bias in the same way will shift the curve towards left. For the case of upper specification limit the effect is just opposite. Imprecision flattens the OC curve, making the plan less discriminating between the good and the bad lots. It is because of additional variability introduced due to the measurement errors.

The variable sampling plan design can be modified to account and compensate for the measurement errors present. The compensating plan will give an observed OC curve with error prone environment which would completely overlap the

desired OC curve. The sequential sampling plan for variables is also considered. Again the effect of measurement errors on the performance measures and the analysis for compensating design is presented.

The economic effects of measurement errors are described briefly. It is seen that bias and imprecision can have drastic cost effects if not accounted for.

The assumption of error free inspection just for the sake of mathematical simplicity cannot be justified. The errors should be incorporated in the model building phase of analysis if quality control systems have to be accurately represented.

6.2 Scope for Further Work:

The present work on 'Inspection errors and sampling plans' is extensive but far from what may be called as complete. In order to join the unbridged gaps the treatment in this thesis needs to be extended in the following directions.

1. Developing a suitable computer program to obtain the computational results based on the model for economic analysis of double sampling plan. This would probably strengthen the analysis given.

Developing some heuristic to design a double sampling plan based on the model presented for economic analysis of

double sampling plan. This would require some adaptive approach and subjective information has to be incorporated.

3. Developing some heuristic to design a single sampling plan based upon AOQL and minimum ATI requirements. A heuristic starting from the plan designed with assumption of perfect inspection may work.

4. Developing a model for the exact economic analysis of a single sampling plan. This will be a simple modification of the model presented for double sampling plan, however, computational results have to be obtained.

5. Developing the analysis of the variables sampling plan in the presence of errors for a more realistic case of unknown variance.

6. Developing analysis for sequential sampling when testing of mean is being considered with two sided alternative or testing of variance of the population is being considered.

7. Developing the analysis for the case when inspection errors are linear function or some higher degree function of the incoming quality. In the present work it has been assumed that inspection error is independent of other factors.

8. Developing the analysis of measurement errors and control charts on the same lines.

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APPENDIX A

Here, we would find expression for $\frac{d\,Pa_e}{dp_e}$ required for computation of ATI_e in case of sequential sampling plan. The following equations were discussed in Sec. 4.2.

$$p'_e = \frac{w_e^{\,s} - 1}{w_e - 1} \qquad (A.1)$$

for $p_e < s$

$$Pa'_e = \frac{w_e^{\,h_1+h_2} - w_e^{\,h_1}}{w_e^{\,h_1+h_2} - 1} \qquad (A.2)$$

$$p''_e = \frac{w_e - w_e^{\,1-s}}{w_e - 1} \qquad (A.3)$$

for $p_e > s$

$$Pa''_e = \frac{w_e^{\,h_1} - 1}{w_e^{\,h_1+h_2} - 1} \qquad (A.4)$$

where $p_e = p(1 - e_2) + (1 - p)\, e_1$

Differentiating (A.1) and (A.2) with respect to w_e, we get,

$$\frac{d\,p'_e}{d\,w_e} = \frac{(w_e-1)\, s\, w_e^{\,s-1} - (w_e^{\,s} - 1)}{(w_e - 1)^2} \qquad (A.5)$$

$$\frac{d\, Pa_e'}{d\, w_e} = \frac{1}{(w_e^{h_1+h_2}-1)^2} [(w_e^{h_1+h_2}-1)\{(h_1+h_2)\, w_e^{h_1+h_2-1}$$

$$- h_1\, w_e^{h_1-1}\} - (w_e^{h_1+h_2} - w_e^{h_1})(h_1+h_2) w_e^{h_1+h_2-1}]$$

(A.6)

$$\frac{d\, Pa_e'}{d\, p_e'} = \frac{d\, Pa_e'/d\, w_e}{d\, p_e'/d\, w_e} \qquad (A.7)$$

Fig. (A.7) is for the case when $p_e < s$. Further, we differentiate Eqs. (A.3) and (A.4) with respect to w_e, to get,

$$\frac{d\, p_e''}{d\, w_e} = \frac{(w_e-1)\,\{1-(1-s)\, w_e^{-s}\} - (w_e - w_e^{1-s})}{(w_e - 1)^2}$$

$$= \frac{s\, w_e^{1-s} + (1-s)\, w_e^{-s} - 1}{(w_e - 1)^2} \qquad (A.8)$$

$$\frac{d\, Pa_e''}{d\, w_e} = \frac{1}{(w_e^{h_1+h_2} - w_e^{h_1})^2} [(w_e^{h_1+h_2} - w_e^{h_1})\, h_1\, w_e^{h_1-1}$$

$$- (w_e^{h_1}-1)\,\{(h_1+h_2) w_e^{h_1+h_2-1} - h_1\, w_e^{h_1-1}\}] \qquad (A.9)$$

$$\frac{d\, Pa_e''}{d\, p_e''} = \frac{d\, Pa_e''/\, d\, w_e}{d\, p_e''/\, d\, w_e} \qquad (A.10)$$

Eq. (A.10) is for the case when $p_e > s$.

For a given incoming quality p, apparent incoming quality p_e is easily obtained. If $p_e < s$, then using Eq. (A.1) and some numerical method we find appropriate w_e such that Eq. (A.1) is satisfied. If $p_e > s$, we find w_e to satisfy Eq. (A.3) in same way. After obtaining w_e Eq. (A.5) and (A.6) or Eqs. (A.8) and (A.9) can be evaluated depending on whether $p_e < s$ or $p_e > s$. Using the above results,

$$\frac{d\,Pa'_e}{d\,p'_e} \quad \text{or} \quad \frac{d\,Pa''_e}{d\,p''_e} \qquad \text{for } p_e < s \text{ and } p_e > s$$

respectively are obtained.

It may be recalled that Eqs. (A.1) to (A.4) are in determinate for $p_e = s$. In such a case following approximate procedure can be used. Find $\frac{d\,Pa_e}{d\,p_e}$ for two values of p that give $p_e = s + \delta$ and $p_e = s - \delta$, where δ is small quantity. The average of the two values so found gives approximate value of $\left.\frac{d\,Pa_e}{d\,p_e}\right|_{p_e=s}$.

APPENDIX B

Here it will be shown that OC curve for the compensating sequential plan in case of measurement errors completely overlaps the desired OC curve. Following equations for OC curve for a compensating plan are from Sec.5.4.

$$K_e = \frac{2\,(s_e - \mu_o)}{\sigma_o^{\,2}} \tag{B.1}$$

$$t_{1e} = h_{1e}\,K_e$$

$$t_{2e} = (h_{1e} + h_{2e})\,K_e \tag{B.2}$$

$$Pa_e = \frac{e^{t_{1e}} - 1}{e^{t_{2e}} - 1} \tag{B.3}$$

where $\mu_o = \mu_p + \mu_e$, $\sigma_o^{\,2} = \sigma^2 + \sigma_e^{\,2}$

$$K_e = \frac{2\,(s_e - \mu_p - \mu_e)}{(\sigma^2 + \sigma_e^{\,2})}$$

$$t_{1e} = h_{1e}\,K_e$$

Substituting,

$$h_{1e} = \frac{b(\sigma^2 + \sigma_e^{\,2})}{\mu_2 - \mu_1} \quad \text{and}$$

$$s_e = \frac{\mu_1 + \mu_2}{2} + \mu_e$$

$$t_{1e} = \frac{b(\sigma^2 + \sigma_e^{\,2})}{\mu_2 - \mu_1} \cdot \frac{2(\frac{\mu_1+\mu_2}{2} + \mu_e - \mu_p - \mu_e)}{(\sigma^2 + \sigma_e^{\,2})}$$

$$= \frac{b}{\mu_2 - \mu_1} \cdot \frac{2(\frac{\mu_1+\mu_2}{2} - \mu_p)}{1}$$

$$= \frac{b\sigma^2}{\mu_2 - \mu_1} \cdot \frac{2(\frac{\mu_1+\mu_2}{2} - \mu_p)}{\sigma^2}$$

Simplifying using,

$$s = \frac{\mu_1 + \mu_2}{2}, \quad h_1 = \frac{b\sigma^2}{\mu_2 - \mu_1}, \quad K = \frac{2(s - \mu_p)}{\sigma^2}$$

$$t_{1e} = h_1 . K = t_1$$

In same way we can get $h_{2e} K_e = h_2 K$, and hence,

$$t_{2e} = (h_{1e} + h_{2e}) K_e = (h_1 + h_2) K = t_2$$

$$Pa_e = \frac{e^{t_{1e}} - 1}{e^{t_{2e}} - 1} = \frac{e^{t_1} - 1}{e^{t_2} - 1} = Pa$$

That is the observed OC curve (Pa_e) for a compensating sequential sampling plan completely overlaps desired OC curve (Pa).